

A Fast Approximation Method of Maximum Operation in Statistical Static Timing Analysis for Achieving Specified Yield

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Abstract— In this paper, we propose a fast maximum-operation of two normal distributions that reduces the error of the estimated worst delay of a circuit obtained by repeating the proposed operation. The proposed maximum-operation outputs a normal distribution by which the worst delay defined is equal to the worst delay defined by the actual distribution and whose shape near the worst delay is close to the actual distribution. In experiments by using benchmark circuits, it is shown that the estimated worst delay obtained by using the proposed method is more accurate than that by existing methods.

I. INTRODUCTION

In designing integrated circuits, it is necessary to confirm the maximum delay of a circuit to guarantee the correct operation. Although static timing analysis (STA) is widely used to estimate the maximum delay of a circuit, the variance of an element delay becomes large according to the manufacturing process advances, and the maximum delay estimated by STA becomes too pessimistic since the possibility that all elements in a path take the maximum delay at the same time is low. Therefore, statistical static timing analysis (SSTA) that treats a delay value as a random variable is actively researched recently [1].

In order to get a high performance circuit with low cost, the yield of a circuit is better to be estimated as accurate as possible. The yield of a circuit is determined by a delay value which is given as a specification and the actual delay distribution of the circuit. In our problem, the yield of a circuit is given as a specification, and the delay value at which the specified yield is achieved is requested to be estimated in short time. In the following, the delay value at which specified yield is achieved is called the worst delay.

In SSTA, given the delay of each element with a distribution, the delay to an element from the primary inputs is obtained as a distribution by using sum-operation and maximum-operation. Sum-operation is used to get a new distribution from two distributions which are in series. While, maximum-operation is used to get a new distribution from distributions which are merged to. Although there exists a method that does not assume a specific distribution [2], normal distribution is usually used as the in-

puts and the output of sum-operation and max-operation to ease the calculation. Fortunately, the output of sum-operation of two normal distributions is a normal distribution. However, the output of max-operation of two normal distributions is not. Therefore, the various methods that approximate the output of maximum-operation to a normal distribution are proposed.

In the paper [3], a method that calculates n -th moment of the output of maximum-operation of two normal distributions was introduced. The mean and variance of the output correspond to the first and second moments, respectively. Although the output of max-operation of two normal distributions is not a normal distribution, a method that calculates the mean and the variance of the output and that approximates the output to the normal distribution with the obtained mean and variance has been used. The outline of the approximated normal distribution obtained by this method is roughly similar to the actual distribution. However, the shape near foot of the approximated normal distribution is not. The worst delay estimated from the approximated normal distribution tends to be smaller than the actual worst delay and a specified yield can not be necessarily achieved.

In this paper, we propose a fast maximum-operation of two normal distributions that reduces the error of the estimated worst delay of a circuit defined by the approximated delay distribution of the circuit obtained by repeating the proposed operation. The proposed maximum-operation outputs a normal distribution by which the worst delay defined is equal to the worst delay defined by the actual distribution and whose shape near the worst delay is close to the actual distribution. In the proposed method, the delay value that achieves a specified yield in the actual distribution is obtained by a numerical calculation. Then, the normal distribution by which the worst delay defined is equal to the worst delay defined by the actual distribution and whose gradient at the calculated worst delay is equal to the actual distribution is selected as the output. In experiments using Monte Carlo simulation with benchmark circuits, it is shown that the estimated worst delay of a circuit by the proposed method is more accurate than that by the existing methods.

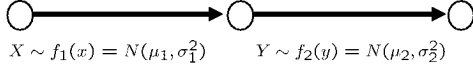


Fig. 1. Sum of the amount of statistics

II. UNIT OPERATIONS OF SSTA

In this paper, we model a circuit by a circuit graph whose edges have delay distributions. By repeating sum-operation and maximum-operation over delay distributions, the delay distribution of the entire circuit is estimated finally. In the following, sum-operation and maximum-operation are explained.

Let X and Y be random variables which correspond to normal distribution $f_1(x) = N(\mu_1, \sigma_1^2)$ and $f_2(y) = N(\mu_2, \sigma_2^2)$, respectively. $N(\mu, \sigma^2)$ is the normal distribution with mean μ and variance σ^2 . Let ρ be the correlation coefficient between X and Y . Let $g(x, y)$ be a bivariate normal distribution of X and Y . Note that $g(x, y)$ includes correlation coefficient ρ . Let z be an arbitrary variable. Let $F_1(x)$ and $F_2(y)$ be cumulative distribution functions of $f_1(x)$ and $f_2(y)$, respectively. $F_1(x)$ and $F_2(y)$ are given as follows.

$$F_1(x) = \int_{-\infty}^x f_1(t)dt \quad (1)$$

$$F_2(y) = \int_{-\infty}^y f_2(t)dt \quad (2)$$

First, we consider sum-operation of two distributions. A sum-operation, which is used when two edges are serially connected as shown in Figure 1, outputs the sum of two distributions. Here, we derive the probability density function $f_{\text{sum}}(z; f_1, f_2)$ that the new random variable $Z = X + Y$ follows. Then $f_{\text{sum}}(z; f_1, f_2)$ is given as follows.

$$f_{\text{sum}}(z; f_1, f_2) = \int_{-\infty}^{\infty} g(x, z-x)dx \quad (3)$$

It is known that Eq.(3) is normal distribution which has mean μ_{sum} and variance σ_{sum}^2 , where μ_{sum} and σ_{sum}^2 are given as follows.

$$\mu_{\text{sum}} = \mu_1 + \mu_2 \quad (4)$$

$$\sigma_{\text{sum}}^2 = \sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2 \quad (5)$$

Next, we consider maximum-operation of two distributions. A maximum-operation, which is used when two edges are parallelly connected as shown in Figure 2, outputs the maximum of two distributions. We derive the probability density function $f_{\text{max}}(z; f_1, f_2)$ that new random variable $Z = \max(X, Y)$ follows. Let $P(Z < z)$ be the probability at which Z is smaller than z , then $P(Z < z)$ is given as follows.

$$\begin{aligned} P(Z < z) &= P(X < z) \cap P(Y < z) \\ &= \int_{-\infty}^z \int_{-\infty}^z g(x, y)dx dy \end{aligned} \quad (6)$$

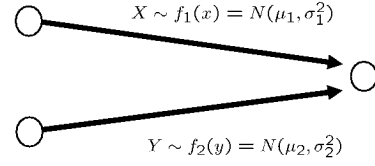


Fig. 2. Maximum of the amount of statistics

Now, $P(Z < z)$ is

$$P(Z < z) = \int_{-\infty}^z f_{\text{max}}(t; f_1, f_2)dt. \quad (7)$$

$f_{\text{max}}(z; f_1, f_2)$ is given by differentiating both sides of Eqs.(6) and (7) with z as follows.

$$f_{\text{max}}(z; f_1, f_2) = \frac{d}{dz} \int_{-\infty}^z \int_{-\infty}^z g(x, y)dx dy \quad (8)$$

If f_1 and f_2 are independent, then $g(x, y) = f_1(x) \cdot f_2(y)$, and $f_{\text{max}}(z; f_1, f_2)$ is derived by

$$f_{\text{max}}(z; f_1, f_2) = F_1(z)f_2(z) + f_1(z)F_2(z). \quad (9)$$

If there is a correlation between f_1 and f_2 , the analytical calculation of Eq.(8) is difficult.

Though the distribution of Eq.(8) is not normal distribution, $f_{\text{max}}(z; f_1, f_2)$ is often approximated to a normal distribution since the correlation between distributions can be easily considered by using a correlation coefficient and the amount of the operation is small, compared with other probability density functions. In the paper [3], a calculation method of n -th moment of $f_{\text{max}}(z; f_1, f_2)$ is given. The mean μ_{max} and the variance σ_{max}^2 of $f_{\text{max}}(z; f_1, f_2)$ are obtained as follows.

$$\mu_{\text{max}} = \mu_1 \cdot \Phi(\beta) + \mu_2 \cdot \Phi(-\beta) + \alpha \cdot \phi(\beta) \quad (10)$$

$$\begin{aligned} \sigma_{\text{max}}^2 &= (\mu_1^2 + \sigma_1^2) \cdot \Phi(\beta) + (\mu_2^2 + \sigma_2^2) \cdot \Phi(-\beta) \\ &\quad + (\mu_1 + \mu_2) \cdot \alpha \cdot \phi(\beta) - \mu_{\text{max}}^2, \end{aligned} \quad (11)$$

where

$$\alpha = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho} \quad (12)$$

$$\beta = \frac{\mu_1 - \mu_2}{\alpha} \quad (13)$$

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \quad (14)$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x \exp\left(-\frac{y^2}{2}\right) dy \quad (15)$$

$$\sigma_1 - \sigma_2 \neq 0 \text{ and } \rho \neq 1 \quad (16)$$

The method that approximates $f_{\text{max}}(z; f_1, f_2)$ to $N(\mu_{\text{max}}, \sigma_{\text{max}}^2)$ has been used in general. We call this method *moment method* here.

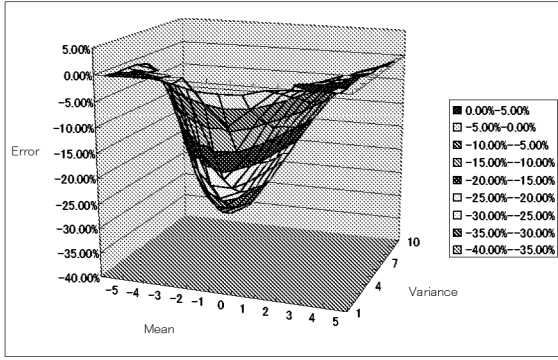


Fig. 3. The estimation error of the maximum-operation by moment method.

III. ERROR OF MAXIMUM OPERATION IN TERMS OF THE WORST DELAY

For a given delay distribution and a specified yield, the value at which a cumulative probability of the distribution becomes the specified yield is defined as the worst value of the distribution. For the normal distribution with mean μ and variance σ^2 , the value that corresponds to $\mu + 3\sigma$ is often used as the worst value of the distribution. This corresponds to the situation that 99.865% is given as a specified yield. For a normal distribution, $\mu + 6\sigma$ might be used as the worst value when a high yield is necessary, and μ might be used as the worst value when a high clock frequency is pursued.

The moment method introduced in the previous section approximates the output of maximum-operation to a normal distribution so that the shape of the approximated distribution is close to the shape of the actual output distribution. However, when the approximated normal distribution is used to estimate the worst delay, the estimated worst delay is often much different from the actual worst delay. That is, the yield achieved by the estimated worst delay is often much different from the specification. When the estimated worst delay is smaller than the actual worst delay, the specified yield is not achieved and the manufacturing cost increases. When the estimated worst delay is larger than the actual worst delay, the design of a high performance circuit becomes difficult. To achieve a high performance in a low manufacturing cost, it is desired that the worst delay value that achieves a specified yield is estimated as accurate as possible.

Now, we consider the maximum-operation of two independent normal distributions. One is fixed by the standard normal distribution of $N(0, 1)$, and the other is $N(-5 \sim 5, 1 \sim 10)$. The errors of the estimated worst delays by moment method when 99.865% yield is specified are shown in Figure 3. The actual worst delay in each case is obtained by the Monte Carlo simulation.

In all cases, the estimated worst delay is underestimated, and the largest error of the estimated worst delays is -38.26% which is caused by the maximum-operation of $N(0 \sim 1)$ and $N(-3, 10)$. In Figure 4, the actual and approximated distributions of maximum-operation of

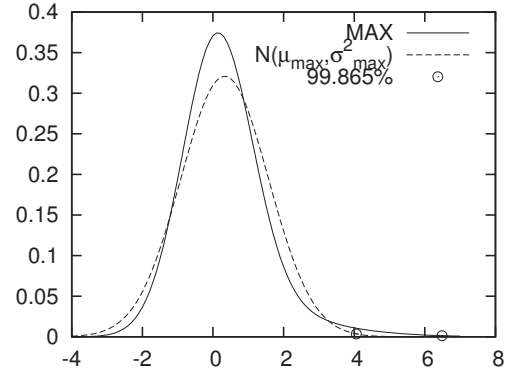


Fig. 4. The actual and approximated distributions of maximum-operation of $N(0, 1)$ and $N(-3, 10)$

$N(0 \sim 1)$ and $N(-3, 10)$ are shown.

This is an analysis of a single maximum-operation. In the estimation of circuit delay, these operations are repeated. Although the error of a single operation might be canceled in repetition, an effort to reduce the final error is required in approximating the output of a single operation.

IV. PROPOSED METHOD

We propose a method of maximum-operation that reduces the error of the estimated worst delay of the output. The proposed method approximates the output of maximum operation to normal distribution $N(\mu'_{\max}, \sigma'^2_{\max})$. In the proposed method, first, the worst delay value \tilde{z} of the actual output is obtained by a numerical method. Then, the proposed method defines μ'_{\max} and σ'_{\max} so that the worst delay defined by $N(\mu'_{\max}, \sigma'^2_{\max})$ is equal to \tilde{z} and the shape of $N(\mu'_{\max}, \sigma'^2_{\max})$ near \tilde{z} is close to the actual output.

First, the worst delay value \tilde{z} is obtained as follows. The cumulative distribution function $F_{\max}(z; f_1, f_2)$ of $f_{\max}(z; f_1, f_2)$ is given from Eqs.(6) and (7) as follows.

$$\begin{aligned}
 F_{\max}(z; f_1, f_2) &= \int_{-\infty}^z f_{\max}(t; f_1, f_2) dt \\
 &= \int_{-\infty}^z \int_{-\infty}^z g(x, y) dx dy \quad (17)
 \end{aligned}$$

Though it is difficult to calculate the cumulative distribution function of bivariate normal distribution, the approximation can be possible by using the method in [6] etc. Note that if two normal distributions are independent of each other, following equation is satisfied since $g(x, y) = f_1(x) \cdot f_2(y)$.

$$\begin{aligned}
 F_{\max}(z; f_1, f_2) &= \int_{-\infty}^z \int_{-\infty}^z f_1(x) f_2(y) dx dy \\
 &= F_1(x) F_2(y). \quad (18)
 \end{aligned}$$

To derive the worst delay value \tilde{z} that achieves specified yield p , the equation of z by which the value of

$F_{\max}(z; f_1, f_2)$ becomes p needs to be solved.

$$F_{\max}(z; f_1, f_2) = p \quad (19)$$

In our proposed method, a numerical calculation method is used to solve Eq.(19) since it is not easy to solve it analytically.

Next, μ'_{\max} and σ'_{\max} are defined as follows. Since the worst delay defined by $N(\mu'_{\max}, \sigma'^2_{\max})$ is equal to \tilde{z} , μ'_{\max} and σ'_{\max} should satisfy the following equation.

$$\mu'_{\max} + n\sigma'_{\max} = \tilde{z} \quad (20)$$

where n is a parameter of achieving specified yield p by standard normal distribution $N(0, 1)$. Since the number of variables is two and the number of equations is one, we still have a flexibility in defining μ'_{\max} and σ'_{\max} . In order to reduce the error in the circuit delay, μ'_{\max} and σ'_{\max} are defined so that shape of $N(\mu'_{\max}, \sigma'^2_{\max})$ near \tilde{z} is close to the actual output. That is, they are defined so that the gradients of the distributions at the worst delay are equal. The gradient of $f(x)$ at $\mu'_{\max} + n\sigma'_{\max}$ of normal distribution $f(x) = N(\mu'_{\max}, \sigma'^2_{\max})$ is calculated as follows.

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma'_{\max}} \exp\left(-\frac{(x - \mu'_{\max})^2}{2\sigma'^2_{\max}}\right) \quad (21)$$

$$f'(x) = -\frac{x - \mu'_{\max}}{\sqrt{2\pi}\sigma'^3_{\max}} \exp\left(-\frac{(x - \mu'_{\max})^2}{2\sigma'^2_{\max}}\right) \quad (22)$$

$$f'(\mu'_{\max} + n\sigma'_{\max}) = -\frac{n}{\sqrt{2\pi}\sigma'^2_{\max}} \exp\left(-\frac{n^2}{2}\right) \quad (23)$$

Next, we derive σ'_{\max} to equate Eq.(23) with $f'_{\max}(\tilde{z}; f_1, f_2)$. That is, we derive σ'_{\max} as follows.

$$-\frac{n}{\sqrt{2\pi}\sigma'^2_{\max}} \exp\left(-\frac{n^2}{2}\right) = f'_{\max}(\tilde{z}; f_1, f_2) \quad (24)$$

$$\sigma'_{\max} = \sqrt{\frac{-n \exp\left(-\frac{n^2}{2}\right)}{\sqrt{2\pi}f'_{\max}(\tilde{z}; f_1, f_2)}} \quad (25)$$

Note that if f_1 and f_2 are independent, $f'_{\max}(z; f_1, f_2)$ is given as follows.

$$f'_{\max}(z; f_1, f_2) = F_1(z)f'_2(z) + 2f_1(z)f_2(z) + f'_1(z)F_2(z) \quad (26)$$

However, if there is correlation between f_1 and f_2 , numerical differentiation of Eq.(17) is used because it is difficult to calculate directly.

Summarizing above description, we proposed method approximates $f_{\max}(z; f_1, f_2)$ to normal distribution $N(\mu'_{\max}, \sigma'^2_{\max})$ deriving from Eqs.(20) and (25).

V. SIMULATIONS

In this section, the experiments with artificial data and the ISCAS89 benchmark circuits to evaluate the proposed method are shown. In order to evaluate the proposed method, we used moment method and the following three methods in comparison.

A. Mean Adjustment Method and Standard Deviation Adjustment Method

In the paper [4], a method which estimates the cumulative probability of $f_{\max}(z; f_1, f_2)$ to become p by the value in which the following equation is satisfied is proposed.

$$\beta' = \max(\mu_1 + n\sigma_1, \mu_2 + n\sigma_2) \quad (27)$$

μ_{\max} and σ_{\max} obtained by the moment method is adjusted so that the worst delay of the output normal distribution is equal to β' .

$$\mu'_{\max} = \beta' - n\sigma_{\max} \quad (28)$$

$$\sigma'_{\max} = \frac{\beta' - \mu_{\max}}{n} \quad (29)$$

In the *mean adjustment method*, μ_{\max} is changed by using Eq.(28). While, in the *standard deviation adjustment method*, σ_{\max} is changed by using Eq.(29).

B. CDF Method

In the paper [5], a method of adjusting σ_{\max} derived by the moment method is proposed as follows. Assume that the random variables X and Y follow normal distributions $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, respectively. Let $P_X(t)$ and $P_Y(t)$ be probabilities that both X and Y are larger than a certain value t , respectively. The probability $P(t)$ that $\max(X, Y)$ is larger than t is given as follows.

$$P(t) = P_X(t) + P_Y(t) - q(t) \quad (30)$$

Note that $q(t)$ is a probability that both X and Y are larger than t . The approximation of t that satisfies $1 - P(t) = p$ is given by the following equation. If $\mu_1 + n\sigma_1 > \mu_2 + n\sigma_2$,

$$t = \mu_1 + n\sigma_1 \frac{-n}{\text{CDF}^{-1}(P(\mu_1 + n\sigma_1))} \quad (31)$$

Note that CDF^{-1} is inverse-function of the cumulative distribution function of standard normal distribution. If $\mu_1 + n\sigma_1 < \mu_2 + n\sigma_2$, t is defined by the Eq.(31) in which the affixes are exchanged. Standard deviation σ'_{\max} is adjusted from this t as follows.

$$\sigma'_{\max} = \frac{t - \mu_{\max}}{n} \quad (32)$$

We call this method in which $N(\mu_{\max}, \sigma'^2_{\max})$ is used as the approximation of $f_{\max}(z; f_1, f_2)$ *CDF method*.

C. Experiment by Artificial Data

SSTA methods in this paper approximate the output of the maximum-operation to normal distribution, and the output is given to an input of next operations. Here, we verify the error of the output of the maximum-operations when a specified yield p is 0.99865.

First of all, we experiment in the circuit graph shown in Figure 5 to examine the error in the repetition of the maximum-operation. In this example, correlation coefficient of the normal distribution of input 1 and input 2 is

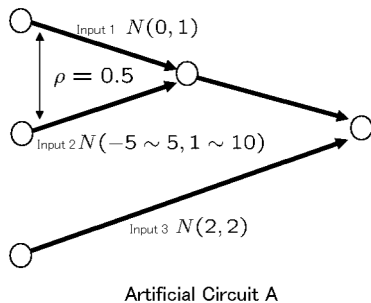


Fig. 5. Repeating maximum-operation

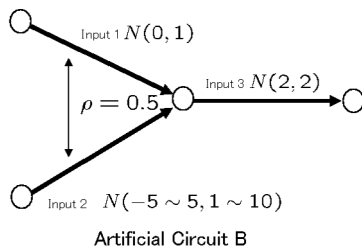


Fig. 6. Sum-operation after maximum-operation

TABLE I
AVERAGE OF ABSOLUTE WORST ESTIMATION ERROR.

	Mom	Mean	Std	Cdf	Pro
Circuit A (%)	9.70	0.68	0.68	0.25	0.15
Circuit B (%)	4.37	2.81	1.19	1.16	0.68

Mom represents Moment method, Mean represents mean adjustment method, Std represents standard deviation adjustment method, Cdf represents CDF method, and Pro represents proposed method.

set to $\rho = 0.5$. Input 1 is fixed by the standard normal distribution of $N(0, 1)$, and input 2 is $N(-5 \sim 5, 1 \sim 10)$. Input 3 which is independent from input 1 and input 2 is $N(2, 2)$.

First, the output of the maximum-operation is approximated to a normal distribution by each method. Second, maximum-operation between input 3 and the approximate normal distribution is done, and the worst delay is estimated by each method. It is assumed that the value estimated by the Monte Carlo simulation is the true worst delay value. The average of absolute error of each method is shown in Table I. The error of each output by the CDF method and the proposed method are shown in Figures 7 and 8, respectively.

Next, the circuit graph shown in Figure 6 is used to examine the error when the sum-operation was applied to the output of the maximum-operation. Input 1, input 2, and input 3 are the same to the above settings. After approximation of maximum-operation between input 1 and input 2, sum-operation between the approximate distribution and input 3 is done. The worst delay value is estimated by each method. This result is shown in Table I. The error of each output by the CDF method and the

TABLE II
PARAMETER USED TO EXPERIMENT

	NOT	AND	NAND	OR	NOR
Average	10.00	16.00	12.00	16.00	12.00
Variance	1.25	2.00	1.33	2.00	1.33

TABLE III
AVERAGE OF YIELD ERROR AND COMPUTING TIME BY ISCAS89 BENCHMARK CIRCUITS(40 CIRCUITS)

	Mom	Mean	Std	Cdf	Pro
Average of negative error (%)	-0.157	-0.139	-0.137	-0.011	-0.016
Average of positive error (%)	0.003	0.000	0.040	0.032	0.032
Average of absolute error (%)	0.157	0.135	0.133	0.037	0.026
Computing time of s38584.1(s)	0.17	0.19	0.18	0.20	0.23

Mom represents Moment method, Mean represents average correct method, Std represents standard deviation correct method, Cdf represents CDF method, and Pro represents proposed method.

proposed method are shown in Figures 9 and 10, respectively.

The average of absolute error by proposed method is the smallest, and the effectiveness of the proposed method was confirmed. As shown in Figures 9 and 10, if the mean of input 2 is smaller than the mean of input 1 and the variance of input 2 is several times larger than the variance of input 1, the error when the sum-operation was done afterwards is large. This trend is common to other existing methods though only the CDF method and the proposed method are shown here due to the limitation of the space.

D. Experiment by Benchmark Data

To show the effectiveness of the proposed method in a real circuit, 40 circuits of the ISCAS89 benchmark are used. The delay is assumed to be caused at a gate of the circuit. The delays by a wire and a register are set to 0. The correlation coefficient of reconvergence paths is estimated by the method proposed in the paper [7]. The parameter used in the experiment is shown in Table II. Newton's method was used as a numeric calculation method from which \tilde{z} was derived. The convergence judgment constant is set to 10^{-5} . $p = 0.99865$ is used. The distribution of the worst delay between registers of each circuit is derived by the Monte Carlo simulation of 100,000 times, and the delay distribution is assumed to be the true distribution. And, the worst value 99.865% point of the distribution was compared with the value of $\mu + 3\sigma$ of each method. The average of yield error in all circuits and the computing time of s38584.1 (the number of vertices is 41,397 and the number of edges is 55,203) whose computing time is the largest are shown in Table III.

The yield error of the proposed method has become small in average. When we pay attention to average of negative yield error, that of CDF method is the smallest in all methods. From the view point of achieving specified yield, CDF method is the best method. However, the average of positive and absolute yield error of CDF method is larger than the proposed method. Therefore, from the view point of the closeness of specified yield, the proposed method is the best method. The computing time of the proposed method is about 1.4 times that of moment method. It seems that the computing time is

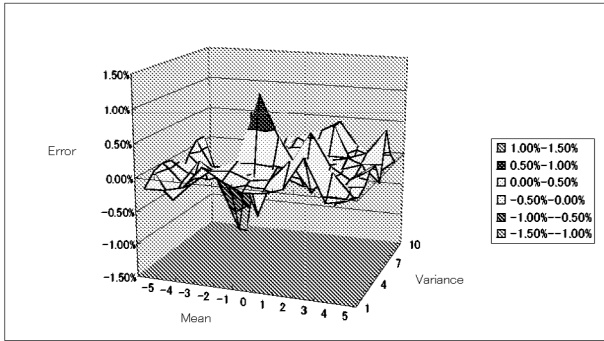


Fig. 7. CDF method(artificial circuit A)

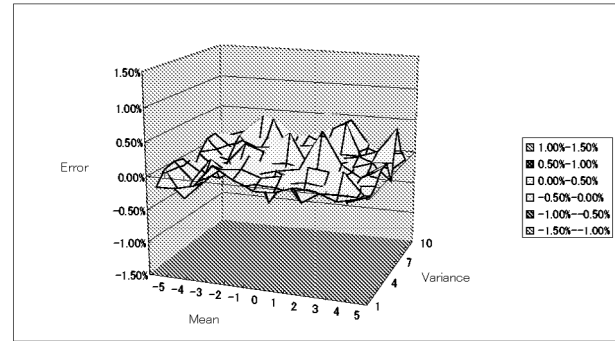


Fig. 8. Proposed method(artificial circuit A)

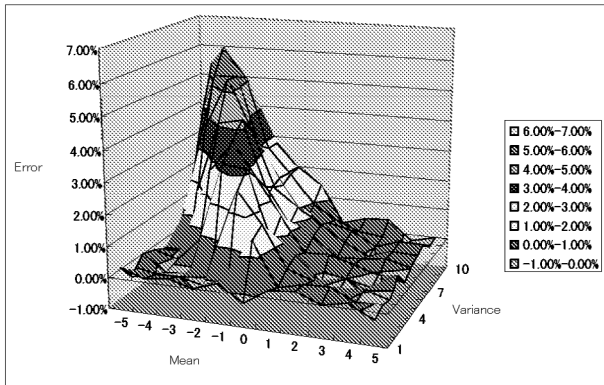


Fig. 9. CDF method(artificial circuit B)

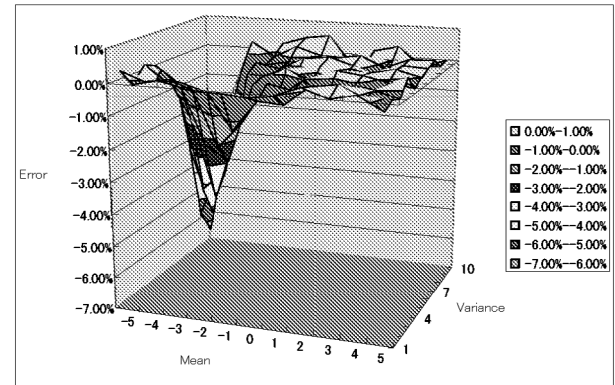


Fig. 10. Proposed method(artificial circuit B)

small enough.

VI. CONCLUSIONS

In this paper, we proposed the method that approximates the output of maximum-operation to normal distribution from the viewpoint of achieving a specified yield. We compare the proposed method with the existing methods by the experiment that uses artificial data and the benchmark data. It was shown that the proposed method was able to reduce the estimated worst delay error in average. The development of the method for improving the worst estimation when sum-operation is done after the maximum-operation and etc. are in our future works.

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