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WDMネットワークにおけるルーティングと波長変換

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あらまし　小文では，波長変換器を備えたノードを持つ波長多重光ネットワークについて考察する．まず，波長変換器を備えたノード数を最小化する問題がNP困難であることを示す単純な証明を与える。また，tree of rings など実用的なグラフにおいては，この問題が多項式時間で解けることを示す．次に，与えられた通信要求を実現するのに必要な波長の数を最小化する問題がtree of rings では多項式時間で解けることを示す．また，近似アルゴリズムについても言及する．

キーワード　波長多重光ネットワーク，ルーティング，波長変換，最大負荷，NP困難

Routing and Wavelength Translation in WDM Networks

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Abstract This paper considers wavelength division multiplexing (WDM) networks with wavelength translators located at nodes. We first give a simple proof of the NP-hardness of the problem of minimizing the number of nodes with wavelength translators which allow the network to run at maximum capacity. Many tractable cases including trees of rings are also mentioned. We next show that the problem of minimizing the number of wavelength used to set up communications is solvable in polynomial time for trees of rings. Some approximation results are also mentioned.

key words　WDM networks, routing, wavelength translation, congestion, NP-hard

1 Introduction

All-optical networks are networks using optical transmission and maintaining optical datapaths through the nodes. Wavelength division multiplexing (WDM) is used to enhance the capacity of the network. In WDM networks, in order to set up a number of communications between given pairs of nodes, routes (called lightpaths) are chosen and wavelengths are assigned to each link along each lightpath so that no two lightpaths going through the same link use the same wavelength along the link. Since the wavelength is a scarce resource, it is a fundamental problem to minimize the number of wavelengths used to set up the communications.

WDM networks are classified roughly into two categories: wavelength selective (WS) networks and wavelength interchanging (WI) networks. In WS networks, the links in a lightpath must be assigned the same wavelength, whereas in WI networks wavelength translators located at nodes are used to change wavelengths along lightpaths and the links in a lightpath may be assigned different wavelengths. For WS networks, the problem of minimizing the number of wavelengths is formulated as the minimum path coloring problem and has been extensively studied in the literature. (See [1, 3, 6, 7, 8, 9, 15] for example.) For WI networks, the problem is reduced to the minimum congestion routing problem which is NP-hard in general.

We show in Section 4 that the minimum congestion routing problem can be solved in polynomial time for the tree of rings, which is a popular real world topology. We make use of results on rings by Wilfong and Winkler [13]. Some approximation results are also mentioned. In Section 3, we consider the problem of minimizing the number of wavelength translators. More precisely, the problem is to minimize the number of nodes with wavelength translators which allow the network to run at maximum capacity. We give a simple proof to show that the problem is NP-hard in general, which was proved in [13]. Many tractable cases including trees of rings are also considered.

2 Definitions

Let G be a graph, and let $V(G)$ and $E(G)$ denote the vertex and edge set of G, respectively. $\delta_G(v)$ is the degree of $v \in V(G)$, and $\Delta(G)$ is the maximum degree of a vertex in G. If D is a directed graph (digraph), we denote the arc set of D by $A(D)$.

The network N_G of a graph G is the digraph obtained from G by replacing each edge e of G with two oppositely oriented arcs with the same ends as e. A routing R_G on N_G is a collection of directed paths (dipaths) on N_G. Let

$$\mathcal{P}_e(R_G) = \{P \in R_G \mid e \in A(P)\}.$$

Define

$$\nu(e, R_G) = |\mathcal{P}_e(R_G)|$$

and

$$\nu(R_G) = \max_{e \in A(N_G)} \nu(e, R_G).$$

$\nu(R_G)$ is called the congestion of R_G.

A wavelength assignment ϕ_e for R_G on an arc $e \in A(N_G)$ is a one-to-one mapping from $\mathcal{P}_e(R_G)$ to $\{1, 2, \ldots, \nu(R_G)\}$. A wavelength assignment ϕ for R_G is defined as

$$\phi = \{\phi_e \mid e \in A(N_G)\}.$$

ϕ for R_G is valid with respect to $S \subset V(N_G)(= V(G))$ if for every vertex $v \notin S$ and every dipath $P \in R_G$ containing arcs (u, v) and (v, w), $\phi_{(u,v)}(P) = \phi_{(v,w)}(P)$. S is called a *sufficient set* for G if there exists a valid wavelength assignment for every routing. A sufficient set corresponds to a set of nodes with wavelength translators. Notice also that the number of wavelengths of a valid wavelength assignment for R_G is optimal for R_G since the congestion of R_G is a trivial lower bound for the number of wavelengths.

A tree T is a *spider* if T has at most one vertex of degree at least 3. A vertex v in a tree T is called a *leaf* if $\delta_T(v) = 1$.

A (u, v)-*chain* in a graph G is a (u, v)-path $(u, w_1, w_2, \ldots, w_k, v)$ $(k \geq 0)$ such that $\delta_G(w_i) = 2$ for any $1 \leq i \leq k$ and $\delta_G(u), \delta_G(v) \geq 3$. u and v are the ends of the chain.

For a graph G and $S \subset V(G)$, define the graph $G(S)$ as follows [13]: $V(G(S))$ consists of the vertices in $V \backslash S$ together with pairs $\langle s, e \rangle$ for each $s \in S$ and each edge e incident to s in G; $E(G(S))$ consists of the edges $\{u, v\}$ of G, where $u, v \notin S$, together with $\{\langle s, e \rangle, v\}$ whenever $e = \{s, v\}$, where $s \in S$, and $\{\langle s, e \rangle, \langle t, e \rangle\}$ whenever s and t are adjacent vertices of S.

3 Minimum Number of Wavelength Translators

We consider in this section the following problem.

MINIMUM SUFFICIENT SET

INSTANCE: Graph G.

QUESTION: Find a sufficient set for G of minimum cardinality.

The networks which require no wavelength translators at all are characterized as follows independently by Wilfong and Winkler [13] and by Gargano, Hell, and Perennes [5].

Theorem I ([5, 13]) *The empty set is sufficient for G if and only if G is a spider.* ∎

Based on Theorem I, the sufficient set can be characterized as follows.

Theorem II ([13]) *$S \subset V(G)$ is sufficient for G if and only if every component of $G(S)$ is a spider.* ∎

The following theorem is proved in [13] by a somewhat complicated method based on the characterization of sufficient sets in Theorem II.

Theorem 1 MINIMUM SUFFICIENT SET *is NP-hard.* ∎

We will show a simple proof of Theorem 1 based on a new characterization of the sufficient set.

3.1 Proof of Theorem 1

It is easy to see the following two lemmas from the definition of $G(S)$.

Lemma 1 *For any path P and spider G, a graph obtained from P and G by identifying an endvertex of P and a leaf of G is a spider.* ∎

Lemma 2 *If $S \subset V(G)$ is sufficient for G then $S \setminus \{v \in S \mid \delta_G(v) = 1\}$ is also sufficient for G.* ∎

Theorem 2 *For any connected graph G with $\Delta(G) \geq 3$, and any sufficient set S for G, there exists a sufficient set S' for G such that $|S'| \leq |S|$ and $\delta_G(v) \geq 3$ for any $v \in S'$.*

Proof: From Lemma 2, assume without loss of generality that $\delta_G(v) \geq 2$ for every $v \in S$. Let $\chi(S) = |\{v \in S \mid \delta_G(v) = 2\}|$. The proof is by induction of $\chi(S)$.

If $\chi(S) = 0$ then the theorem is true by putting $S' = S$.

Assume that the theorem is true for the case when $\chi(S) = k \geq 0$, and consider the case when $\chi(S) = k+1$ for induction. Let v be a vertex in S with $\delta_G(v) = 2$, let x_v be a nearest vertex to v with $\delta_G(x_v) \geq 3$, and let $y_v \in S$ be a nearest vertex to x_v on the (v, x_v)-path. Consider $S_1 = S \cup \{x_v\}$ and $S_2 = S \setminus \{y_v\} = S \cup \{x_v\} \setminus \{y_v\}$. Since S is sufficient for G, S_1 is also sufficient for G. Let e_1 be the edge incident to y_v in the (x_v, y_v)-path and let e_2 be the other edge incident to y_v. Let G_1 and G_2 be the connected components of $G(S_1)$ containing $\langle y_v, e_1\rangle$ and $\langle y_v, e_2\rangle$, respectively, and let G_3 be the connected component of $G(S_2)$ containing y_v. In order to prove that S_2 is a sufficient set for G, it is sufficient to prove that G_3 is a spider, since the other connected components of $G(S_2)$ are contained in $G(S_1)$, and hence spiders. Note that G_3 is obtained from G_1 and G_2 by identifying $\langle y_v, e_1\rangle$ and $\langle y_v, e_2\rangle$ into one vertex y_v. Thus, G_3 is a spider from Lemma 1, and so S_2 is a sufficient set for G such that $|S_2| \leq |S|$. Note that $\chi(S_2) = k$. Thus, by the induction hypothesis, there exists a sufficient set $S' \subset V(G)$ for G such that $|S'| \leq |S_2| \leq |S|$ and $\delta_G(v) \geq 3$ for any $v \in S'$, which completes the proof. ∎

A sufficient set S for a graph G is said to be *select* if $\delta_G(v) \geq 3$ for any $v \in S$. The following is a direct consequence of Theorem 2.

Theorem 3 *For any connected graph G with $\Delta(G) \geq 3$, there exists a minimum sufficient set for G which is select.* ∎

A connected graph G with $\Delta(G) \leq 2$ is a path or ring. Since a path is a spider by definition, the empty set is sufficient for a path. It is easy to see that a set consisting of just one vertex is sufficient for a ring. Therefore, we consider, in what follows, graphs with maximum degree at least 3.

Theorem 3 allows us to consider only vertices of degree at least 3. We remove other vertices by the following procedure.

Input: Graph G.

Step 1: $P = \{v \in V(G) \mid \delta_G(v) = 1\}$.

Step 2: $Q = \{v \in V(G) \mid \delta_G(v) = 2\}$.

for each $v \in Q$ **begin**

- Let $e_1 = \{v, u_1\}$ and $e_2 = \{v, u_2\}$ be the edges incident to v.
- $E(G) = E(G) \setminus \{e_1, e_2\} \cup \{\{u_1, u_2\}\}$.
- **if** $\delta_G(v) = 0$ **then** $V(G) = V(G) \setminus \{v\}$.

end

Step 3:

for each $v \in P$ **begin**

if $\delta_G(v) = 1$ **then**

$V(G) = V(G) \setminus \{v\}$,

$E(G) = E(G) \setminus \{e\}$, where e is the edge incident to v.

endif

end

Step 4: Remove the parallel edges from $E(G)$, leaving one of them.

Figure 1: Procedure CONTRACT

In step 2, the vertices on all chains and all tails are removed from G. In step 3, the pendant vertices and the edges incident to them in the original graph G are removed from G such that every connected component in G has at least one vertex. The graph obtained from G by procedure CONTRACT is called the *contracted graph* of G, and denoted by G_γ. Notice that the input of procedure CONTRACT is any graph, and so the contracted graph is defined for any graph. In particular, the contracted graph of a path is consisting of just one vertex, and the contracted graph of a ring is consisting of a vertex with a self-loop.

It is easy to see the following lemmas.

Lemma 3 *For any connected graph G with $\Delta(G) \geq 3$, $V(G_\gamma) = \{v \mid \delta_G(v) \geq 3\}$.* ∎

Lemma 4 *Any two vertices u and v in G_γ are connected by an edge if and only if G has a (u,v)-chain.* ∎

Now, we are ready to state our characterization for the select sufficient set. A subset $C \subset V(G)$ is called a *vertex cover* of a graph G if at least one endvertex of every edge belongs to C.

Theorem 4 *For any connected graph G with $\Delta(G) \geq 3$, $S \subset V(G)$ is a select sufficient set for G if and only if S is a vertex cover for G_γ.*

Proof: Let $S \subset V(G)$ be a select sufficient set for G and assume that S is *not* a vertex cover of G. Then, there exists an edge $e = \{u, v\} \in E(G_\gamma)$ such that e is not covered by vertices from S, that is $u, v \notin S$. Thus, we have a (u,v)-chain C in G from Lemma 4. Since S is a select sufficient set, any vertex in C is not in S, and so the vertices in C is contained in a connected component G' of $G(S)$. Note that $\delta_{G'}(u) \geq 3$ and $\delta_{G'}(v) \geq 3$. Thus, if $u \neq v$ then G' is not a spider, which is a contradiction to the assumption that S is a sufficient set for G. Hence we conclude that $u = v$. Then, G' contains a ring as a subgraph and so G' is not a spider, which is a contradiction.

Conversely, let $S \subset V(G_\gamma)$ be a vertex cover for G_γ and assume that S is *not* a select sufficient set. Then, there exists a connected component G' of $G(S)$ such that G' is not a spider. There are two cases to be considered:

(i) G' has at least two vertex with degree at least 3. Then, there exists a (u,v)-chain in G', and so the vertices in the (u,v)-chain in G is not in S. Then, G_γ has the edge $e = \{u, v\}$ such that $u, v \in S$, that is, e is not covered by vertices from S, which is a contradiction.

(ii) G' has at most one vertex with degree at least 3, but a ring R. If G' is a ring then G is disconnected or $\Delta(G) = 2$. Hence G' is not a ring. Since G' has at most one vertex with degree at least 3, the vertex v with $\delta_{G'}(v) \geq 3$ is in R, that is, there exists a (v,v)-chain C in G', and so vertices in G corresponding to C is not in S. Then, G_γ has the self-loop $e = \{v, v\}$ such that $v \notin S$, that is, e is not covered by vertices from S, which is a contradiction. ∎

It is easy to see that for any connected graph G with $\Delta(G) \leq 2$ i.e. path or ring, $S \subset V(G)$ is a minimum sufficient set for G if S is a minimum vertex cover for G_γ. Thus, we have the following theorem from Theorem 4.

Theorem 5 *For any graph G, S is a minimum sufficient set for G if S is a minimum vertex cover for G_γ.* ∎

Now, we are going to complete the proof of Theorem 1. We will show that the following decision version of MINIMUM SUFFICIENT SET:

SUFFICIENT SET

INSTANCE: Graph G and a positive integer $K \le |V(G)|$.

QUESTION: Is there a sufficient set of size K or less for G ?

is NP-complete.

It is easy to see that SUFFICIENT SET belongs to NP.

We will reduce the following problem:

VERTEX COVER

INSTANCE: Graph G and positive integer $K \le |V(G)|$.

QUESTION: Is there a vertex cover of size K or less for G ?

to SUFFICIENT SET. Let G and K be the graph and a positive integer in an arbitrary instance of VERTEX COVER. We construct a graph G' such that there exists a sufficient set S' for G' with $|S'| \le K$ if and only if there exists a vertex cover S for G with $|S| \le K$.

Let G' be the graph obtained from G by adding $3 - \delta_G(v)$ new vertices adjacent to v for each vertex $v \in V(G)$, with $\delta_G(v) \le 2$. G' can be constructed from G in $O(|V(G)|)$ time.

Consider the contracted graph G'_γ of G'. Let $V_0 = \{v \in V(G') \mid \delta_{G'}(v) = 1\}$. Note that $\delta_{G'}(v) \ge 3$ for any $v \in V(G') \backslash V_0$. Also note that the subgraph of G' induced by $V(G')\backslash V_0$ is the graph G. Thus, G'_γ is identical to G from Lemma 3.

Let $S \subset V(G)(= V(G'_\gamma))$ be a vertex cover for G with $|S| \le K$. Then, S is a select sufficient set for G' with $|S| \le K$ from Theorem 4.

Conversely, let $S \subset V(G')$ be a sufficient set for G' with $|S| \le K$. From Theorem 2, there exists a select sufficient set S' for G' such that $|S'| \le |S| \le K$. Then, S' is a vertex cover for $G(= G'_\gamma)$ with $|S'| \le K$ from Theorem 4.

Since VERTEX COVER is a well-known NP-complete problem [4], SUFFICIENT SET is also NP-complete. ∎

3.2 Tractable Cases

Let $\tau(G)$ denote the cardinality of a minimum vertex cover for G. For any $V' \subset V(G)$, let $G[V']$ denote the subgraph of G induced by V'.

Lemma 5 *If $(V_1, \ldots, V_k)$ is a partition of $V(G)$,*

$$\tau(G) \ge \sum_{i=1}^{k} \tau\left(G[V_i]\right).$$

Sketch of Proof: Since, for any V_i $(1 \le i \le k)$, all edges in $E(G[V_i])$ should be covered by vertices from V_i, the right hand side of the inequality is a lower bound for $\tau(G)$. ∎

3.2.1 Trees

It is easy to see the following lemma from the definition of a contracted graph.

Lemma 6 *G_γ is a tree if G is a tree.* ∎

Theorem 6 MINIMUM SUFFICIENT SET *is solvable in polynomial time for trees.*

Proof: Since a minimum vertex cover can be found in polynomial time for bipartite graphs [14] and a tree is a bipartite graph, MINIMUM SUFFICIENT SET is solvable in polynomial time for trees from Theorem 5 and Lemma 6. ∎

3.2.2 Partial k-trees

Let K_n be the complete on n vertices. The class of *k-trees* is defined recursively as follows [15]:

1. K_k is a k-tree.
2. If G is a k-tree and k vertices $v_1, v_2, \ldots, v_k$ induce a complete subgraph of G, then the graph G' defined as $V(G') = V(G) \cup \{w\}$ and $E(G') = E(G) \cup \{\{v_i, w\} \mid 1 \le i \le k\}$ is a k-tree where w is a new vertex not contained in G.
3. All k-trees can be formed with rules (1) and (2).

A graph is a *partial k-tree* if it is a subgraph of a k-tree.

A graph H is a *minor* of a graph G if H is isomorphic to a graph obtained from a subgraph of G by contracting edges. A contracted graph of G is a minor of G from the definition of contracted graphs. Since a minor of a partial k-tree is a partial k-tree, we obtain the following lemma.

Lemma 7 *For any partial k-tree G, the graph obtained from G_γ by removing all self-loops is a partial k-tree.* ∎

The following lemma can be found in [2].

Lemma 8 *A minimum vertex cover can be found in polynomial time for partial k-trees.* ∎

Theorem 7 MINIMUM SUFFICIENT SET *solvable in polynomial time for partial k-trees.*

Proof: Let G be a partial k-tree, and let V_0 be the set of vertices in G_γ with self-loops. Then, $\tau(G_\gamma[V_0]) = |V_0|$. Thus, $\tau(G_\gamma) \geq \tau(G_\gamma[V(G)\backslash V_0]) + |V_0|$ from Lemma 5.

Let S' be a minimum vertex cover for $G_\gamma[V(G)\backslash V_0]$. Since $G_\gamma[V(G)\backslash V_0]$ is a partial k-tree from Lemma 7, S' can be found in polynomial time. Consider $S = S' \cup V_0$. It is easy to see that S is a vertex cover for G_γ. Since $|S'| = \tau(G_\gamma[V(G)\backslash V_0])$, we conclude that $|S| = |S'| + |V_0| \leq \tau(G_\gamma)$. Hence S is a minimum vertex cover for G_γ, and so S is a minimum sufficient set for G from Theorem 5. ∎

3.2.3 Series-Parallel Graphs

A graph G is *homeomorphic* with a graph H if G and H can be obtained from the same graph by subdividing edges.

G is a *series-parallel graph* if no subgraph of G is homeomorphic with K_4. Since a series-parallel graph is a partial 2-tree [2], the following corollary is obtained from Theorem 7.

Corollary 1 MINIMUM SUFFICIENT SET *is solvable in polynomial time for series-parallel graphs.* ∎

3.2.4 Trees of Rings

A graph G is a *tree of rings* if any two rings in G share at most one vertex and the graph obtained from G by contracting the edges in the rings is a tree. Since a tree of rings is a series-parallel graph as easily seen, we have the following.

Corollary 2 MINIMUM SUFFICIENT SET *is solvable in polynomial time for trees of rings.* ∎

3.2.5 Meshes

An $N \times M$ *mesh* $M_{N\times M}$ is defined as follows:

$$\begin{aligned} V(M_{N\times M}) &= \{0,\ldots,N-1\} \times \{0,\ldots,M-1\} \\ E(M_{N\times M}) &= \{\{(x,y),(p,q)\} \mid \\ &\qquad (x,y),(p,q) \in V(M_{N\times M}), \\ &\qquad |x-p|+|y-q| = 1\}. \end{aligned}$$

It is easy to verify the following lemma.

Lemma 9 *If G is an n-vertex mesh,* $\tau(G) = \lfloor \frac{n}{2} \rfloor$. ∎

Let M_N be an $N \times N$ mesh. For $N \geq 3$, the contracted graph $(M_N)_\gamma$ is obtained from M_N by replacing every corner vertex and the edges incident to it with a single edge. We denote here $(M_N)_\gamma$ by W_N.

Theorem 8 MINIMUM SUFFICIENT SET *is solvable in polynomial time for* M_N *with* $N \geq 3$. *Moreover, If S be a minimum sufficient set for* M_N,

$$|S| = \begin{cases} 3 & \text{if } N = 3, \\ \left\lfloor \frac{N^2}{2} \right\rfloor & \text{if } N \geq 4. \end{cases}$$

Proof: If $N = 3$, it is easy to see that $S = \{(1,0),(1,1),(1,2)\}$ is a minimum sufficient set.

Assume $N \geq 4$. Let $U_0 = \{0,1\}$, $U_1 = \{2,3,\ldots,N-3\}$, and $U_2 = \{N-2,N-1\}$. For any $i,j \in \{0,1,2\}$, define $V_{i,j} = (U_i \times U_j) \cap V(W_N)$. It is easy to see the following:

1. $G[V_{0,0}]$, $G[V_{0,2}]$, $G[V_{2,0}]$, and $G[V_{2,2}]$ are isomorphic to K_3;

2. $G[V_{0,1}]$, $G[V_{1,0}]$, $G[V_{1,2}]$, and $G[V_{2,1}]$ are isomorphic to the $2 \times (N-4)$ mesh;

3. $G[V_{1,1}]$ is isomorphic to the $(N-4) \times (N-4)$ mesh.

Thus, from Lemmas 5 and 9,

$$\begin{aligned} \tau(W_N) &\geq 4\cdot 2 + 4\cdot(N-4) + \left\lfloor \frac{(N-4)^2}{2} \right\rfloor \\ &= \left\lfloor \frac{N^2}{2} \right\rfloor. \end{aligned}$$

Conversely, consider a minimum vertex cover U for M_N, that is, $U = V(M_N) \cap M_\infty$, where

$$\begin{aligned} M_\infty = \{(x,y) \mid y = x+k \\ \text{s.t. } x,y \in Z, k \in \{\pm 1, \pm 3, \pm 5, \ldots\}\}. \end{aligned}$$

U covers all edges in $E(M_N) \cap E(W_N)$.

The following notation is used: $v_{\swarrow} = (0,0)$, $v_{\swarrow}^{\rightarrow} = (1,0)$, $v_{\swarrow}^{\uparrow} = (0,1)$, $v_{\searrow} = (N-1,0)$, $v_{\searrow}^{\leftarrow} = (N-2,0)$, $v_{\searrow}^{\uparrow} = (N-1,1)$, $v_{\nwarrow} = (0,N-1)$, $v_{\nwarrow}^{\rightarrow} = (1,N-1)$, $v_{\nwarrow}^{\downarrow} = (0,N-2)$, $v_{\nearrow} = (N-1,N-1)$, $v_{\nearrow}^{\leftarrow} = (N-2,N-1)$, and $v_{\nearrow}^{\downarrow} = (N-1,N-2)$. Then, W_N is the graph obtained from M_N by replacing $v_{\swarrow}, v_{\searrow}, v_{\nwarrow}, v_{\nearrow}$ and the edges incident to them with a single edge per every corner respectively.

If N is odd, four edges in $E(W_N)\backslash E(M_N)$, i.e. $\{v_{\swarrow}^{\uparrow}, v_{\swarrow}^{\rightarrow}\}$, $\{v_{\searrow}^{\uparrow}, v_{\searrow}^{\leftarrow}\}$, $\{v_{\nwarrow}^{\downarrow}, v_{\nwarrow}^{\rightarrow}\}$, and $\{v_{\nearrow}^{\downarrow}, v_{\nearrow}^{\leftarrow}\}$ are covered by vertices from U because all endvertices of the edges are in U. Note that $v_{\nwarrow}, v_{\searrow} \notin U$. Thus, U is a vertex cover for W_N if N is odd.

On the other hand, if N is even, four edges in $E(W_N)\backslash E(M_N)$ are not covered by vertices from U because all endvertices of the edges are not in U. Note that $v_{\nwarrow}, v_{\searrow} \in U$. In this case,

$$U' = U\backslash\{v_{\nwarrow}, v_{\searrow}\} \cup \{v_{\nwarrow}^{\rightarrow}, v_{\searrow}^{\leftarrow}\}$$

is a vertex cover for W_N if N is even.

Therefore,

$$\begin{aligned} |U| &= \{2+4+\cdots+(N-1)\} \times 2 \\ &= \frac{1}{2} \times (N+1) \times \frac{N-1}{2} \times 2 \\ &= \frac{N^2-1}{2} \quad (N \text{ is odd}), \end{aligned}$$

and

$$\begin{aligned} |U'| &= \{1+3+\cdots+(N-1)\} \times 2 \\ &= \frac{1}{2} \times N \times \frac{N}{2} \times 2 \\ &= \frac{N^2}{2} \quad (N \text{ is even}). \end{aligned}$$

Thus, if S is a minimum vertex cover for W_N,

$$|S| \leq \left\lfloor \frac{N^2}{2} \right\rfloor,$$

which shows an upper bound for $|S|$. Since both bounds are equal, U and U' are a minimum vertex cover for W_N with odd and even N, respectively. U and U' can be computed in $O(N^2)$. ∎

4 Minimum Number of Wavelengths

Let N_G be a WI networks, and

$$\pi = \{(s_i,t_i) \mid s_i,t_i \in V(N_G), 1 \leq i \leq k\}$$

be a set of node pairs (s_i,t_i) to be connected by a dipath from s_i to t_i ((s_i,t_i)-dipath). A set of dipaths

$$R_G(\pi) = \{P_i \mid P_i \text{ is an } (s_i,t_i)\text{-dipath}, 1 \leq i \leq k\}$$

is called a routing of π. The congestion $\nu(R_G(\pi))$ of $R_G(\pi)$ is the maximum number of dipaths in $R_G(\pi)$ using an arc of N_G, as define in Section 2. If N_G has wavelength translators on nodes of a sufficient set, for any routing $R_G(\pi)$ of π, there exists a valid wavelength assignment using $\nu(R_G(\pi))$ wavelengths, which is optimal for $R_G(\pi)$. Thus the problem of setting up the communications for π with the minimum number of wavelengths is reduced to the following problem.

MINIMUM CONGESTION ROUTING

INSTANCE: Network N_G and a set of node pairs $\pi = \{(s_i,t_i) \mid s_i,t_i \in V(N_G), 1 \leq i \leq k\}$.

QUESTION: Find a routing of π with minimum congestion.

The following is the only known nontrivial solvable case for MINIMUM CONGESTION ROUTING, as far as the authors know.

Theorem III ([13]) MINIMUM CONGESTION ROUTING *is solvable in polynomial time for rings.* ∎

Theorem III can be easily extended to trees of rings. Suppose N_G is a tree of rings, and R is a ring in N_G. If an (s_i,t_i)-dipath P_i passes through R, let p_i and q_i be the first and last vertices of P_i which is in R, respectively. Let π_R be the set of such pairs (p_i,q_i). Since all possible (s_i,t_i)-dipaths contain both p_i and q_i, the problem of minimizing the congestion on R is reduced to MINIMUM CONGESTION ROUTING with R and π_R as the instance. Thus, we have the following from Theorem III.

Theorem 9 MINIMUM CONGESTION ROUTING *is solvable in polynomial time for trees of rings.* ∎

Raghavan and Thompson [10] showed general approximation algorithms for MINIMUM CONGESTION ROUTING using the multicommodity flow relaxation and the randomized rounding scheme. Kleinberg [6] mentioned without proof that there exists a constant-factor approximation algorithm for two-dimensional meshes. An extensive survey of the approximation of MINIMUM CONGESTION ROUTING can be found in [11]. Among other things, it is shown that if the congestion is relatively large ($\Omega(\log|A(N_G)|)$), there exists a constant-factor approximation algorithm for any network.

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