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A Note on the Three-Dimensional Channel Routing

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1 Introduction

This paper shows a generalization of a previous result on the 3-D channel routing presented in [2].

In the 3-D channel routing, the channel is a 3-D grid G consisting of columns, rows, and layers which are planes defined by fixing x-, y-, and z-coordinates, respectively. A terminal is a vertex of G located in the top or bottom layer. A net is a set of terminals to be connected. A net containing k terminals is called a k-net. A tree connecting the terminals in a net is called a wire. The object of the 3-D channel routing problem is to connect the terminals in each net with a wire in G using as few layers as possible and as short wires as possible in such a way that wires for distinct nets are disjoint. The number of layers is called the height of the 3-D channel. The following two theorems are shown in [2].

Theorem I *If the layers are square 2-D grids of area $4n$, the terminals are located on vertices with odd x- and y-coordinates, and each net has terminals both in top and bottom layers, then any set of n 2-nets can be routed in a 3-D channel of height $\mathcal{O}(\sqrt{n})$ using wires of length $\mathcal{O}(\sqrt{n})$.*

Theorem II *There exists a set of n 2-nets that requires a 3-D channel of height $\Omega(\sqrt{n})$ to be routed.*

Theorem I implies that any set of n 2-nets can be routed in a 3-D channel of volume $\mathcal{O}(n^{3/2})$, while for the ordinary 2-D channel routing there exists a set of n 2-nets requiring a 2-D channel of area $\Omega(n^2)$ to be routed [1].

The purpose of this paper is to show a generalization of Theorem I as follows:

Theorem 1 *If the layers are square 2-D grids of area $(\sqrt{2n}+1)^2$, the terminals are located on vertices with odd x-coordinates, and each net has terminals both in top and bottom layers, then any set of n 2-nets can be routed in a 3-D channel of height $\mathcal{O}(\sqrt{n})$ using wires of length $\mathcal{O}(\sqrt{n})$.*

2 Preliminaries

We consider a 3-D channel of height $h+1$, which is a $(\sqrt{2n}+1)\times(\sqrt{2n}+1)\times(h+1)$ 3-D grid. Each grid point is denoted by (x,y,z) with $0 \le x,y \le \sqrt{2n}$ and $0 \le z \le h$. The column, row, and layer defined by $x=i$, $y=j$, and $z=k$ are called the i-column, j-row, and k-layer, respectively. (See Fig. 1.) The h-layer and 0-layer are corresponding to the top and bottom layers, respectively. Let $\mathcal{N}=\{N_i | 0 \le i \le n-1\}$ be a set of n 2-nets, and let $(X_i^{(h)}, Y_i^{(h)}, h)$ and $(X_i^{(0)}, Y_i^{(0)}, 0)$ be the terminals of N_i $(0 \le i \le n-1)$, where $X_i^{(h)}$ and $X_i^{(0)}$ are odd for every i, $Y_i^{(h)}, Y_i^{(0)} \ge 1$ for every i, and $(X_i^{(h)}, Y_i^{(h)}, h) \ne (X_j^{(h)}, Y_j^{(h)}, h)$ and $(X_i^{(0)}, Y_i^{(0)}, 0) \ne (X_j^{(0)}, Y_j^{(0)}, 0)$ if $i \ne j$.

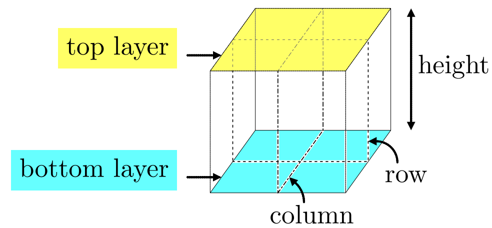

Figure 1: The three-dimensional channel.

3 Sketch of the Proof of Theorem 1

The theorem is proved by showing a polynomial time routing algorithm. The algorithm consists of three phases, each of which uses $\mathcal{O}(\sqrt{n})$ layers. We use two virtual terminals $(X_i^{(l)}, Y_i^{(l)}, l)$ and $(X_i^{(m)}, Y_i^{(m)}, m)$ for each net N_i such that $X_i^{(l)} = X_i^{(h)}$ and $X_i^{(m)} = X_i^{(0)}$, and such that $Y_i^{(l)} \ne Y_j^{(l)}$ if $X_i^{(h)} = X_j^{(h)}$ for $i \ne j$, where $0 < m < l < h$. Such virtual terminals can be computed in polynomial time as shown in [2]. We connect $(X_i^{(h)}, Y_i^{(h)}, h)$ with $(X_i^{(l)}, Y_i^{(l)}, l)$ in the first phase, connect $(X_i^{(l)}, Y_i^{(l)}, l)$ with $(X_i^{(m)}, Y_i^{(m)}, m)$ in the second phase, and connect $(X_i^{(m)}, Y_i^{(m)}, m)$ with $(X_i^{(0)}, Y_i^{(0)}, 0)$ in the last phase. In each phase, the connection of terminals is accomplished by using a polynomial time 2-D channel routing algorithm proposed in [1].

The second phase consists of three steps. We use two more virtual terminals $(X_i^{(l-1)}, Y_i^{(l-1)}, l-1)$ and $(X_i^{(m+1)}, Y_i^{(m+1)}, m+1)$ for each net N_i such that $X_i^{(l-1)} = X_i^{(l)}$ and $Y_i^{(l-1)} = Y_i^{(l)}$ if $X_i^{(l)}$ is odd, $X_i^{(l-1)} = X_i^{(l)}+1$ and $Y_i^{(l-1)} = Y_i^{(l)}-1$ if $X_i^{(l)}$ is even, $X_i^{(m+1)} = X_i^{(m)}$ and $Y_i^{(m+1)} = Y_i^{(m)}$ if $X_i^{(m)}$ is odd, and $X_i^{(m+1)} = X_i^{(m)}+1$ and $Y_i^{(m+1)} = Y_i^{(m)}-1$ if $X_i^{(m)}$ is even. We connect $(X_i^{(l)}, Y_i^{(l)}, l)$ and $(X_i^{(l-1)}, Y_i^{(l-1)}, l-1)$ within a subgrid consisting of $(l-1)$- and l-layers in the first step, connect $(X_i^{(l-1)}, Y_i^{(l-1)}, l-1)$ and $(X_i^{(m+1)}, Y_i^{(m+1)}, m+1)$ within a subgrid consisting of layers between $(l-1)$- and $(m+1)$-layers in the second step, and connect $(X_i^{(m+1)}, Y_i^{(m+1)}, m+1)$ and $(X_i^{(m)}, Y_i^{(m)}, m)$ within a subgrid consisting of $(m+1)$- and m-layers in the last step.

References

[1] R.Y. Pinter, "On routing two-point nets across a channel," 19th Design Automation Conference, pp.894–902, 1982.

[2] S. Tayu, P. Hurtig, Y. Horikawa, and S. Ueno, "On the three-dimensional channel routing," IPSJ SIG Technical Report, 2004-AL-98, pp.106–117, 2004.