A-1-21

## A Note on the Three-Dimensional Single-Active-Layer Routing

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## 1 Introduction

The 3-D channel is a 3-D grid G consisting of columns, rows, and layers which are planes defined by fixing x-, y-, and z-coordinates, respectively. The numbers of columns, rows, and layers are called the width, depth, and height of G, respectively. G is called a (W, D, H)channel if the width is W, depth is D, and height is HA vertex of G is a grid point with integer coodinates. We assume without loss of generality that the vertex set of a (W, D, H)-channel is  $\{(x, y, z)|x \in [W], y \in$  $[D], z \in [H]\}$ , where  $[n] = \{1, 2, ..., n\}$  for a positive integer n. Layers defined by z = H and z = 1 are called the top and bottom layers, respectively. (See Fig.1.)

A terminal is a vertex of G located on the top or bottom layer. A net is a set of terminals to be connected. A net containing k terminals is called a k-net. A k-net is called a multiterminal net if  $k \ge 2$ . A tree connecting the terminals in a net is called a wire. The object of the routing problem is to connect the terminals in each net with a wire using as few layers as possible and as short wire as possible in such a way that wires for distinct nets are vertex-disjoint. In case of the 3-D single-active-layer routing all the terminals are located either on the top or bottom layer. This is a special case of the 3-D channel routing where each terminal is located on the top or bottom layer. A set of nets is said to be routable in G if G has vertex-disjoint trees spanning the nets.

For the 3-D channel routing, it has been known that the problem of deciding if a given set of nets is routable in a 3-D channel is NP-complete [1]. It has been also known that if G is a (2N, 2N, 3N)-channel, the terminals are located on vertices with odd x- and y-coordinates, and each net has terminals both on top and bottom layers, then any set of 2-nets is routable in G [2]. Moreover, if G is a  $(2N, 2N, N/2 - \epsilon)$ -channel for any  $\epsilon > 0$ , the terminals are located on vertices with odd x- and y-coordinates, and each net has terminals both on top and bottom layers, then there exists a set of 2-nets that is not routable in G [2].

For the 3-D single-active-layer routing, it has been known that if G is a (2N, 2N, 3N)-channel, and the terminals are located on vertices with odd x- and ycoordinates, then any set of 2-nets is routable in G [3]. It has been also known that if G is a (2N, 2N, 6N)channel, the terminals are located on vertices with odd x- and y-coordinates, then any set of multiterminal nets is routable in G [3]. Moreover if G is a  $(2N, 2N, N/4 - \epsilon)$ -channel for any  $\epsilon > 0$ , and the terminals are located on vertices with odd x- and ycoordinates, then there exists a set of 2-nets that is not routable in G [3].

The purpose of this paper is to improve results above as follows.

**Theorem 1** If G is a (2N, 2N, 5N/2)-channel, and the terminals are located on vertices with odd x- and y-coordinates either on the top or bottom layer, then any set of multiterminal nets is routable in G.



Figure 1: Three-dimensional channel

## 2 Proof Sketch of Theorem 1

The theorem is proved by showing a polynomial time routing algorithm. The algorithm is a modification of a 3-D channel routing algorithm in [2], and consists of three phases. Each of the first two phases uses Nlayers, while the last phase uses at most N/2 layers.

Let  $t_1, t_2, \ldots, t_n$  be the terminals on the top layer. We assume without loss of generality that the terminals in every multiterminal net are consecutive in the sequence. We use virtual terminals  $s_1, s_2, \ldots, s_n$ arranged in a snakelike order on the bottom layer such that  $s_1, s_2, \ldots, s_N$  are on the same column defined by x = 1. We also use virtual 2-nets  $\{t_1, s_1\}, \{t_2, s_2\}, \ldots, \{t_n, s_n\}$ .

We apply the first two phases of 3-D channel routing algorithm in [2] for the virtual 2-nets using 2N layers. Then we can prove from the arrangement of the virtual terminals that the terminals of every original multiterminal net can be connected by using at most N/2 more layers.

## References

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