On the Three-Dimensional Orthogonal Drawing of Outerplanar Graphs (Extended Abstract)

Satoshi Tayu, Takuya Oshima, and Shuichi Ueno
Department of Communications and Integrated Systems, Tokyo Institute of Technology, Tokyo 152-8550-S3-57, Japan

Abstract— **It has been known that every series-parallel 6-graph has a 2-bend 3-D orthogonal drawing, while it has been open whether every series-parallel 6-graph has a 1-bend 3-D orthogonal drawing. We show in this paper that every outerplanar 5-graph has a 1-bend 3-D orthogonal drawing.**

I. Introduction

We consider the problem of generating orthogonal drawings of graphs in the space. The problem has obvious applications in the design of 3-D VLSI circuits and optoelectronic integrated systems [3], [5].

Throughout this paper, we consider simple connected graphs G with vertex set $V(G)$ and edge set $E(G)$. We denote by $d_G(v)$ the degree of a vertex v in G, and by $\Delta(G)$ the maximum degree of a vertex of G. G is called a k-graph if $\Delta(G) \leq k$. The connectivity of a graph is the minimum number of vertices whose removal results in a disconnected graph or a single vertex graph. A graph is said to be k-connected if the connectivity of the graph is at least k.

It is well-known that every graph can be drawn in the space so that its edges intersect only at their ends. Such a drawing of a graph G is called a *3-D drawing* of G. A graph is said to be planar if it can be drawn in the plane so that its edges intersect only at their ends. Such a drawing of a planar graph G is called a *2-D drawing* of G.

A *3-D orthogonal drawing* of a graph G is a 3-D drawing such that each edge is drawn by a sequence of contiguous axis-parallel line segments. Notice that a graph G has a 3-D orthogonal drawing only if $\Delta(G) \leq 6$. A 3-D orthogonal drawing with no more than b bends per edge is called a b-bend 3-D orthogonal drawing.

Eades, Symvonis, and Whitesides [2], and Papakostas and Tollis [6] showed that every 6-graph has a 3-bend 3-D orthogonal drawing. Eades, Symvonis, and Whitesides [2] also posed an interesting open question of whether every 6-graph has a 2-bend 3-D orthogonal drawing. Wood [8] showed that every 5-graph has a 2-bend 3-D orthogonal drawing. Tayu, Nomura, and Ueno [7] showed that every series-parallel 6-graph has a 2-bend 3-D orthogonal drawing. Moreover, Nomura, Tayu, and Ueno [4] showed that every outerplanar 6-graph has a 0-bend 3-D orthogonal drawing if and only if it contains no triangle as a subgraph, while Eades, Stirk, and Whitesides [1] proved that it is NP-complete to decide if a given 5-graph has a 0-bend 3-D orthogonal drawing. Tayu, Nomura, and Ueno [7] also posed an interesting open question of whether every series-parallel 6-graph has a 1-bend 3-D orthogonal drawing.

We shown in this paper the following theorem.

Theorem 1: Every outerplanar 5-graph has a 1-bend 3-D orthogonal drawing.

The proof of Theorem 1 is constructive and provides a polynomial time algorithm to generate such a drawing for an outerplanar 5-graph. It is still open whether every series-parallel 6-graph has a 1-bend 3-D orthogonal drawing.

II. Preliminaries

A 2-D drawing of a planar graph G is regarded as a graph isomorphic to G, and referred to as a plane graph. A plane graph partitions the rest of the plane into connected regions. A *face* is a closure of such a region. The unbounded region is referred to as the *external face*. We denote the boundary of a face f of a plane graph Γ by $b(f)$. If Γ is 2-connected then $b(f)$ is a cycle of Γ.

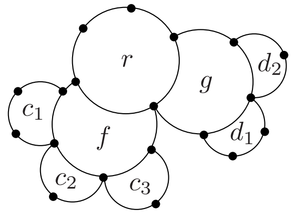
Given a plane graph Γ, we can define another graph Γ^* as follows: corresponding to each face f of Γ there is a vertex f^* of Γ^*, and corresponding to each edge e of Γ there is an edge e^* of Γ^*; two vertices f^* and g^* are joined by the edge e^* in Γ^* if and only if the edge e in Γ lies on the common boundary of faces f and g of Γ. Γ^* is called the *(geometric-)dual* of Γ.

A graph is said to be *outerplanar* if it has a 2-D drawing such that every vertex lies on the boundary of the external face. Such a drawing of an outerplanar graph is said to be *outerplane*. It is well-known that an outerplanar graph is a series-parallel graph. Let Γ be an outerplane graph with the external face f_o, and $\Gamma^* - f_o^*$ be a graph obtained from Γ^* by deleting vertex f_o^* together with the edges incident to f_o^*. It is easy to see that if Γ is an outerplane graph then $\Gamma^* - f_o^*$ is a forest. In particular, an outerplane graph Γ is 2-connected if and only if $\Gamma^* - f_o^*$ is a tree.

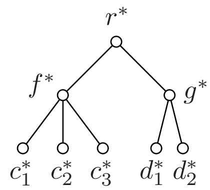
III. 2-Connected Outerplanar Graphs

We first consider the case when G is 2-connected. Let G be a 2-connected outerplanar 5-graph and Γ be an outerplane graph isomorphic to G. Since Γ is 2-connected, $T^* = \Gamma^* - f_o^*$ is a tree. A vertex r^* of T^* is designated as a root, and T^* is considered as a rooted tree. If l^* is a leaf of T^* then l is called a *leaf face* of Γ. If g^* is a child of f^* in T^* then f is called the *parent face* of g, and g is called a *child face* of f in Γ. The unique edge in $b(f) \cap b(g)$ is called the *base* of g. We choose r^* so that $b(r) \cap b(f_o) \neq \emptyset$, and any edge in $b(r) \cap b(f_o)$ is defined as the base of r. Let S^* be a rooted subtree of T^* with root r^*. If S^* is consisting of just r^* then S^* is denoted by r^*. $\Gamma[S^*]$ is a subgraph of Γ induced by the vertices on boundaries of faces of Γ corresponding to the vertices of S^*. It should be noted that $\Gamma[S^*]$ is a 2-connected

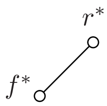
978-1-4244-3828-0/09/$25.00 ©2009 IEEE



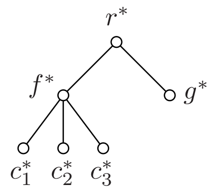
(a) Γ.



(b) T^*.



(c) S^*.



(d) $\overline{S^*}$.

Fig. 1. Example of an outerplanae graph Γ, rooted tree T^*, subtrees S^* and $\overline{S^*}$ of T^*.

outerplane graph. Let f^* be a vertex in $V(T^*) - V(S^*)$ which is a child of a vertex $p^* \in V(S^*)$. $S^* + f^*$ is a subtree of T^* obtained from S^* by adding f^* and edge (f^*, p^*). Let $\overline{S^*}$ be a rooted subtree of T^* with root r^* induced by the vertices of S^* and the children of the vertices of S^*. Fig. 1 shows an example of an outerplane graph Γ, rooted tree T^*, and rooted subtrees S^* and $\overline{S^*}$.

For any face f of Γ, $b(f)$ is a cycle since Γ is 2-connected. Let

$$\begin{aligned} V(b(f)) &= \{u_i \mid 0 \le i \le k-1\}, \text{and} \\ E(b(f)) &= \{(u_0, u_{k-1})\} \cup \\ &\quad \{(u_i, u_{i+1}) \mid 0 \le i \le k-2\}, \end{aligned}$$

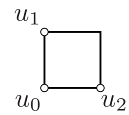
where (u_i, u_{k-1}) is the base of f. A 1-bend 3-D orthogonal drawing of $b(f)$ is said to be *canonical* if $b(f)$ is drawn as one of the following four configurations.

Configuration 1 (Rectangle-1) : If $k = 3$ then only (u_1, u_2) has a bend as shown in Fig. 2(a). If $k \ge 4$ then every edge has no bend, and $u_1, u_2, \ldots, u_{k-2}$ are drawn on a side of a rectangle as shown in Fig. 2(b).

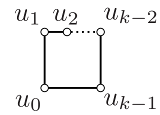
Configuration 2 (Rectangle-2) : If $k = 3$ then every edge has a bend, and u_1 is at a corner of a rectangle as shown in Fig. 2(c). If $k \ge 4$ then only (u_0, u_{k-1}) and (u_0, u_1) have a bend, $u_1, u_2, \ldots, u_{k-2}$ are drawn on a side of a rectangle, and u_0 and u_{k-1} are on another different sides of the rectangle as shown in Fig. 2(d).

Configuration 3 (Hexagon) : If $k = 3$ then every edge has a bend as shown in Fig. 2(e). If $k \ge 4$ then only (u_0, u_{k-1}) and (u_0, u_1) have a bend, and $u_1, u_2, \ldots, u_{k-2}$ are on a side of a hexagon as shown in Fig. 2(f).

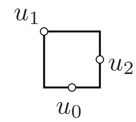
Configuration 4 (Book) : A *book* is obtained from a rectangle by bending at a line segment, called the *spine*, parallel to a side of the rectangle. If $k = 3$ then every edge has a bend as shown in Fig. 2(g). If $k \ge 4$ then only (u_0, u_{k-1}), (u_0, u_1), and (u_{k-2}, u_{k-1}) have a bend, and $u_1, u_2, \ldots, u_{k-2}$ are on a side of a book as shown in Fig. 2(h).



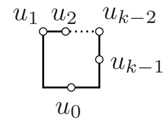
(a) Rectangle-1 for $k = 3$.



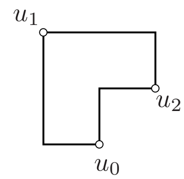
(b) Rectangle-1 for $k \ge 4$.



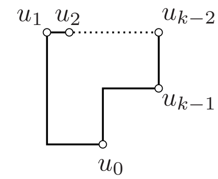
(c) Rectangle-2 for $k = 3$.



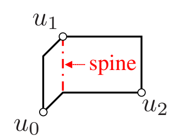
(d) Rectangle-2 for $k \ge 4$.



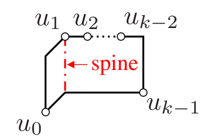
(e) Hexagon for $k = 3$.



(f) Hexagon for $k \ge 4$.

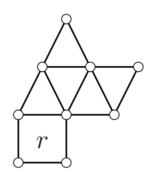


(g) Book for $k = 3$.

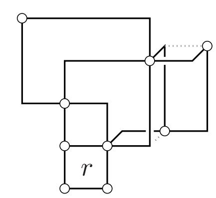


(h) Book for $k \ge 4$.

Fig. 2. Rectangle-1 and -2, Hexagon, and Book.



(a) Γ.



(b) 1-bend 3-D orthogonal canonical drawing of Γ.

Fig. 3. Example of Γ and 1-bend 3-D orthogonal canonical drawing of Γ.

A drawing of Γ is said to be *canonical* if every face is drawn canonically. Fig. 3 shows an example of an outerplane graph Γ, and a 1-bend 3-D orthogonal canonical drawing of Γ.

Roughly speaking, we will show that if $\Gamma[\overline{S^*}]$ has a 1-bend 3-D orthogonal canonical drawing then $\Gamma[\overline{S^* + f^*}]$ also has a 1-bend 3-D orthogonal canonical drawing, where f^* is a leaf of $\overline{S^*}$. The following theorem immediately follows by induction.

Theorem 2: A 2-connected outerplanar 5-graph has a 1-bend 3-D orthogonal drawing.

III-A. Proof of Theorem 2

For a grid point $p = (p_x, p_y, p_z)$ and a vector $\boldsymbol{v} = (v_x, v_y, v_z)$, let $p + \boldsymbol{v}$ be the grid point $(p_x + v_x, p_y + v_y, p_z + v_z)$.

For a unit vector $\boldsymbol{d}$, we denote $-\boldsymbol{d} = \overline{\boldsymbol{d}}$. Define that $\boldsymbol{e}_x = (1, 0, 0)$, $\boldsymbol{e}_y = (0, 1, 0)$, $\boldsymbol{e}_z = (0, 0, 1)$, and $D = \{\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z, \overline{\boldsymbol{e}_x}, \overline{\boldsymbol{e}_y}, \overline{\boldsymbol{e}_z}\}$. Every vector in D is called a *direction*.

A 3-D orthogonal drawing of a plane graph Γ can be regarded as a pair $\langle \phi, \rho \rangle$ of one-to-one mappings $\phi : V(\Gamma) \to \mathbb{Z}^3$ and ρ which maps edges (u, v) to internally disjoint paths on the 3-D grid $\mathcal{G}$ connecting $\phi(u)$ and $\phi(v)$. For a direction $\boldsymbol{d} \in D$ and a vertex $v \in V(\Gamma)$, $\langle \phi, \rho \rangle$ is said to be *$\boldsymbol{d}$-free* at $\phi(v)$ if $\rho(e)$ does not contain the edge of $\mathcal{G}$ connecting $\phi(v)$ and $\phi(v) + \boldsymbol{d}$.

Let Γ be a 2-connected outerplane graph, and $\langle \phi, \rho \rangle$ be a 3-D orthogonal canonical drawing of Γ. Let f be a leaf face of Γ, and

$$V(b(f)) = \{u_i \mid 0 \le i \le k-1\},$$
$$E(b(f)) = \{(u_0, u_{k-1})\} \cup \{(u_i, u_{i+1}) \mid 0 \le i \le k-2\},$$

where (u_0, u_{k-1}) is the base of f. We define three unit vectors $\boldsymbol{d}_0(f, u_0)$, $\boldsymbol{d}_1(f, u_0)$, and $\boldsymbol{d}_2(f, u_0)$ as follows:

- If f is drawn as a rectangle-1, we define that $\boldsymbol{d}_0(f, u_0)$ is the unit vector directed from $\phi(u_{k-1})$ to $\phi(u_0)$, $\boldsymbol{d}_1(f, u_0) = \overline{\boldsymbol{d}_0(f, u_0)}$, and $\boldsymbol{d}_2(f, u_0)$ is a unit vector orthogonal to the rectangle.
- If f is drawn as a rectangle-2, let p be the bend of base (u_0, u_{k-1}). We define that $\boldsymbol{d}_1(f, u_0)$ is a unit vector orthogonal to the rectangle, and $\boldsymbol{d}_0(f, u_0)$ is the unit vector directed from $\phi(u_0)$ to p.
- If f is drawn as a hexagon, let p be the bend of base (u_0, u_{k-1}). We define that $\boldsymbol{d}_0(f, u_0)$ is the unit vector directed from p to $\phi(u_0)$, $\boldsymbol{d}_1(f, u_0)$ is the unit vector directed from p to $\phi(u_{k-1})$. and $\boldsymbol{d}_2(f, u_0)$ is a unit vector orthogonal to the hexagon.
- If f is drawn as a book, let p be the bend of base (u_0, u_{k-1}). We define that $\boldsymbol{d}_0(f, u_0)$ is the unit vector directed from $\phi(u_{k-1})$ to p, $\boldsymbol{d}_1(f, u_0)$ is the unit vector directed from $\phi(u_0)$ to p, and $\boldsymbol{d}_2(f, u_0)$ is the unit vector directed from the bend q of edge (u_{k-2}, u_{k-1}) to $\phi(u_{k-1})$.

A 1-bend 3-D orthogonal canonical drawing $\langle \phi, \rho \rangle$ of Γ is called a *1-bend 3-D orthogonal τ-drawing* of Γ if $\langle \phi, \rho \rangle$ satisfies one of the following conditions for every leaf face f. Let (u_0, u_{k-1}) be the base of a leaf face f.

Condition 1: f is drawn as a rectangle-1 or hexagon, and

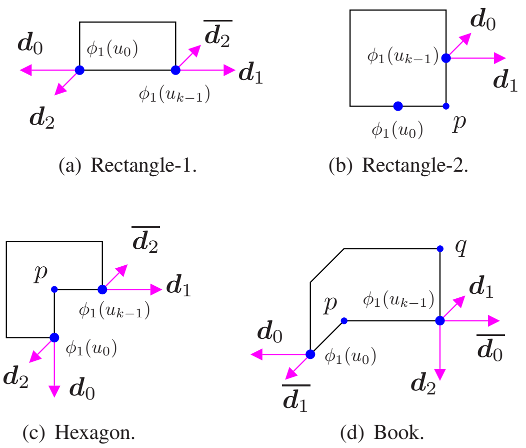
- if $d_\Gamma(u_0) \le 4$ then $\langle \phi, \rho \rangle$ is $\boldsymbol{d}_0(f, u_0)$-free or $\boldsymbol{d}_2(f, u_0)$-free at $\phi(u_0)$,
- if $d_\Gamma(u_{k-1}) \le 4$ then $\langle \phi, \rho \rangle$ is $\boldsymbol{d}_1(f, u_0)$-free or $\overline{\boldsymbol{d}_2(f, u_0)}$-free at $\phi(u_{k-1})$; (See Fig. 4(a) and (c).)

Condition 2: f is drawn as a rectangle-2, and

- $d_\Gamma(u_0) = 5$,
- $\langle \phi, \rho \rangle$ is $\boldsymbol{d}_0(f, u_0)$-free at $\phi(u_{k-1})$,
- if $d_\Gamma(u_{k-1}) \le 3$ then $\langle \phi, \rho \rangle$ is $\boldsymbol{d}_1(f, u_0)$-free at $\phi(u_{k-1})$. (See Fig. 4(b).)

Condition 3: f is drawn as a book, and

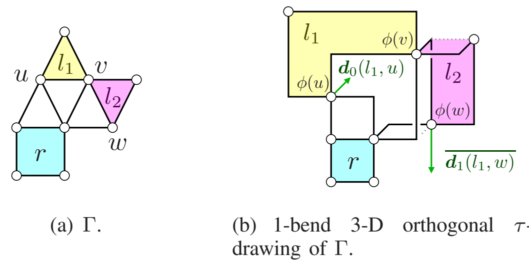
- if $d_\Gamma(u_0) \le 4$ then $\langle \phi, \rho \rangle$ is $\boldsymbol{d}_0(f, u_0)$-free or $\overline{\boldsymbol{d}_1(f, u_0)}$-free at $\phi(u_0)$,



(a) Rectangle-1. (b) Rectangle-2.

(c) Hexagon. (d) Book.

Fig. 4. Directions for draiwng of face f.



(a) Γ. (b) 1-bend 3-D orthogonal τ-drawing of Γ.

Fig. 5. Example of Γ and 1-bend 3-D orthogonal τ-drawing of Γ.

- if $d_\Gamma(u_{k-1}) \le 4$ then $\langle \phi, \rho \rangle$ is $\boldsymbol{d}_1(f, u_0)$-free or $\overline{\boldsymbol{d}_0(f, u_0)}$-free at $\phi(u_{k-1})$,
- if $d_\Gamma(u_0) \le 4$, $d_\Gamma(u_{k-1}) \le 4$, $\langle \phi, \rho \rangle$ is not $\boldsymbol{d}_0(f, u_0)$-free at $\phi(u_0)$, and $\langle \phi, \rho \rangle$ is not $\boldsymbol{d}_1(f, u_0)$-free at $\phi(u_{k-1})$ then $\langle \phi, \rho \rangle$ is $\boldsymbol{d}_2(f, u_0)$-free at $\phi(u_{k-1})$, and $d_\Gamma(u_{k-1}) = 4$,
- spine except for their ends is not used in the drawing; (See Fig. 4(d).)

Fig. 5 shows an example of an outerplane graph Γ, and a 1-bend 3-D orthogonal τ-drawing of Γ. In order to prove Theorem 2, it suffices to prove the following.

Theorem 3: A 2-connected outerplanar 5-graph has a 1-bend 3-D orthogonal τ-drawing.

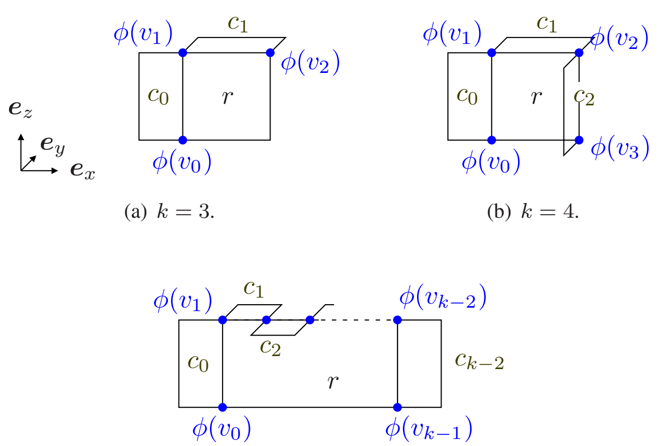
Proof (Sketch): Let G be a 2-connected outerplanar 5-graph, Γ be a outerplane graph isomorphic to G, and $T^* = \Gamma^* - f_o^*$ be a tree rooted at r^*. We prove the theorem by induction. The basis of the induction is stated as follows.

Lemma 1: $\Gamma\overline{[r^*]}$ has a 1-bend 3-D orthogonal τ-drawing.

Proof of Lemma 1: Let

$$V(b(r)) = \{v_i \mid 0 \le i \le k-1\},$$
$$E(b(r)) = \{(v_0, v_{k-1})\} \cup \{(v_i, v_{i+1}) \mid 0 \le i \le k-2\},$$

where (v_0, v_{k-1}) is the base of r. Let c_i be a child face of r with base (v_i, v_{i+1}) for $0 \le i \le k-2$, if any. Let $\langle \phi, \rho \rangle$ be a 1-bend 3-D orthogonal canonical drawing of $\Gamma\overline{[r^*]}$ as shown in Fig. 6, where c_i is drawn as rectangle-1, if any. Since $\langle \phi, \rho \rangle$ is $\boldsymbol{d}_0(c_0, v_0)$-free at $\phi(v_0)$ and $\boldsymbol{d}_1(c_0, v_0)$-free at $\phi(v_1)$, $\langle\phi,\rho\rangle$ satisfies Condition 1 for c_0. If $k = 3$, by taking $\boldsymbol{d}_2(c_1, v_1) = \boldsymbol{e}_z$, $\langle\phi,\rho\rangle$ is $\boldsymbol{d}_2(c_1, v_1)$-free at $\phi(v_1)$ and $\boldsymbol{d}_1(c_1, v_1)$-free at $\phi(v_2)$. Therefore, $\langle\phi,\rho\rangle$ also satisfies Condition 1 for c_1, and we conclude that $\langle\phi,\rho\rangle$ is a 1-bend 3-D orthogonal τ-drawing of $\Gamma[\overline{r^*}]$. If $k = 4$, by similar arguments to $k = 3$, $\langle\phi,\rho\rangle$ also satisfies Condition 1 for c_1. Also by taking $\boldsymbol{d}_2(c_2, v_2) = \boldsymbol{e}_x$, $\langle\phi,\rho\rangle$ is $\boldsymbol{d}_2(c_2, v_2)$-free at $\phi(v_2)$ and $\boldsymbol{d}_1(c_2, v_2)$-free at $\phi(v_3)$. Therefore, $\langle\phi,\rho\rangle$ also satisfies Condition 1 for c_2, and we conclude that $\langle\phi,\rho\rangle$ is a 1-bend 3-D orthogonal τ-drawing of $\Gamma[\overline{r^*}]$. If $k \geq 5$, by taking $\boldsymbol{d}_2(c_i, v_i) = \boldsymbol{e}_z$ for $1 \leq i \leq k-3$, $\langle\phi,\rho\rangle$ is $\boldsymbol{d}_2(c_i, v_i)$-free at $\phi(v_i)$ and $\overline{\boldsymbol{d}_2(c_i, v_i)}$-free at $\phi(v_{i+1})$. Thus, $\langle\phi,\rho\rangle$ satisfies Condition 1 for c_i $(1 \leq i \leq k-3)$. Similarly, by taking $\boldsymbol{d}_2(c_{k-2}, v_{k-2}) = \overline{\boldsymbol{e}_z}$, $\langle\phi,\rho\rangle$ is $\boldsymbol{d}_2(c_{k-3}, v_{k-3})$-free at $\phi(v_{k-3})$ and $\overline{\boldsymbol{d}_2(c_{k-3}, v_{k-3})}$-free at $\phi(v_{k-2})$. Thus, $\langle\phi,\rho\rangle$ satisfies Condition 1 for c_{k-3}. Also, $\langle\phi,\rho\rangle$ satisfies Condition 1 for c_{k-2}, since $\langle\phi,\rho\rangle$ is $\boldsymbol{d}_0(c_{k-2}, v_{k-2})$-free at $\phi(v_{k-2})$ and $\boldsymbol{d}_1(c_{k-2}, v_{k-2})$-free at $\phi(v_{k-1})$. So, we conclude that $\langle\phi,\rho\rangle$ is a 1-bend 3-D orthogonal τ-drawing of $\Gamma[\overline{r^*}]$. ■



(a) $k = 3$. (b) $k = 4$. (c) $k \geq 5$.

Fig. 6. Drawing of initial case.

Let S^* be a rooted subtree of T^* with root r^*. The inductive step is stated as follows.

Lemma 2: If $\Gamma[\overline{S^*}]$ has a 1-bend 3-D orthogonal τ-drawing then $\Gamma[\overline{S^* + f^*}]$ also has a 1-bend 3-D orthogonal τ-drawing, where f^* is a leaf of $\overline{S^*}$.

The proof of Lemma 2 is omitted in the extended abstract due to space limitation.

IV. General Outerplanar Graphs

In this section, we shall complete the proof of Theorem 1. We assume without loss of generality that G is a connected outerplanar 5-graph. A *block* of G is a maximal 2-connected subgraph or a bridge (cut edge). Let $G_1, G_2, \ldots, G_m$ be blocks of G. It is well-known that $E(G)$ can be partitioned into $E(G_1), E(G_2), \ldots,$ and $E(G_m)$. An *adjacency graph* A_G of G is defined as follows: $V(A_G) = \{G_1, G_2, \ldots, G_m\}$, and $(G_i, G_j) \in E(A_G)$ if and only if $V(G_i) \cap V(G_j) \neq \emptyset$. Notice that A_G is connected since G is connected. Suppose $(G_1, G_2, \ldots, G_m)$ is a preorder of $V(A_G)$ obtained by applying DFS on A_G. Then a subgraph H_i of G induced by the vertices in $\bigcup_{k=1}^{i} V(G_k)$ is connected for $1 \leq i \leq m$. We prove Theorem 1 by induction on i. A subgraph $H_1 = G_1$ is a block of G. If H_1 is a 2-connected outerplar 5-graph, we know by Theorem 2 that H_1 has a 1-bend 3-D orthoognal drawing. If H_1 is a bridge, it is easy to see that H_1 has a 0-bend 3-D orthogonal drawing. The inductive step is stated as follows.

Lemma 3: For $1 \leq i \leq m-1$, if H_i has a 1-bend 3-D orthogonal drawing then H_{i+1} has a 1-bend 3-D orthogonal drawing.

This proves Theorem 1 since $H_m = G$. The proof of Lemma 3 is omitted in the extended abstract due to space limitation.

V. Concluding Remarks

It is not possible to directly extend our construction method to outerplanar 6-graphs. Suppose a face is drawn as a book. Then, for some cases, the direction along the spine of the book cannot be used for the drawing of descendant faces of the face. Thus, we can use no more than 5 directions at some grid points.

The proof of Theorem 2 is constructive and provide a polynomial time algorithm to construct 1-bend 3-D orthogonal drawings. For example, a 1-bend 3-D orthogonal drawing shown in Fig. 5(b) can be obtained as follows. For an outerplane graph Γ shown in Fig. 5(a), our algorithm firstly construct a 1-bend 3-D orthogonal drawing of root face r and its child face f_1. Faces r and f_1 are drawn as Rectangle-1 (Fig. 2(a)), since r and f_1 correspond to r and c_0 in the proof of Lemma 1 (Fig. 6(b)). Then the child face f_2 of f_1 is drawn as a hexagon (Fig. 2(e)). Next, the child faces f_3 and f_4 of f_2 are drawn as a hexagon and a book (Fig. 2(g)), respectively. Finally, the child face f_5 of f_4 is drawn as a book, and we obtain a 1-bend 3-D orthgonal drawing of Γ shown in Fig. 5(b).

References

[1] P. Eades, C. Strik, and S. Whitesides, "The techniques of Komolgorov and Bardzin for three-dimensional orthogonal graph drawings," Information Processing Letters, vol.60, pp.97–103, 1996.

[2] P. Eades, A. Symvonis, and S. Whitesides, "Three-dimensional orthogonal graph drawing algorithms," Discrete Applied Mathematics, vol.103, pp.55–87, 2000.

[3] F.T. Leighton and A.L. Rosenberg, "Three-dimensional circuit layouts," SIAM J. Comput., vol.15, no.3, pp.793–813, 1986.

[4] K. Nomura, S. Tayu, and S. Ueno, "On the orthogonal drawing of outerplanar graphs," IEICE Trans. Fundamentals, vol.E88-A, no.6, pp.1583–1588, 2005.

[5] S. Obenaus and T. Szymanski, "Embedding of star graphs into optical meshes without bends," J. of Parallel and Distributed Computing, vol.44, no.2, pp.97–106, 1997.

[6] A. Papakostas and I. Tollis, "Algorithm for incremental orthogonal graph drawing in three dimensions," J. Graph Algorithms and Applications, vol.3, no.4, pp.81–115, 1999.

[7] S. Tayu, K. Nomura, and S. Ueno, "On the three-dimensional orthogonal drawing of series-parallel graphs," Proceedings of 2008 IEEE International Symposium on Circuits and Systems, pp.212-215, 2008.

[8] D. Wood, "Optimal three-dimensional orthogonal graph drawing in the general position model," Theoretical Computer Science, vol.299, no.1–3, pp.151–178, 2003.