The Complexity of Three-Dimensioal Channel Routing (Extended Abstract)

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Abstract: The 3-D channel routing is a fundamental problem on the physical design of 3-D integrated circuits. The 3-D channel is a 3-D grid G and the terminals are vertices of G located in the top and bottom layers. A net is a set of terminals to be connected. The objective of the 3-D channel routing problem is to connect the terminals in each net with a Steiner tree (wire) in G using as few layers as possible and as short wires as possible in such a way that wires for distinct nets are disjoint. This paper shows that the problem is intractable.

Keywords: 3-D channel, bend, net, $\mathcal{NP}$-complete, $(r \times s - 1)$-puzzle, Steiner tree.

1 Introduction

The three-dimensional (3-D) integration is an emerging technology to implement large circuits, and currently being extensively investigated. (See [1, 2, 3, 4, 6, 11, 15, 16] for example.) In this paper, we consider a problem on the physical design of 3-D integrated circuits.

The 3-D channel is a 3-D grid G consisting of *columns*, *rows*, and *layers* which are rectilinear grid planes defined by fixing x-, y-, and z-coordinates at integers, respectively. The numbers of columns, rows, and layers are called the *width*, *depth*, and *height* of G, respectively. (See Fig. 1.) G is called a (W, D, H)-*channel* if the width is W, depth is D, and height is H. A vertex of G is a grid point with integer coordinates. We assume without loss of generality that the vertex set of a (W, D, H)-channel is $\{(x, y, z)|x \in [W], y \in [D], z \in [H]\}$, where $[n] = \{1, 2, \ldots, n\}$ for a positive integer n. Layers defined by $z = H$ and $z = 1$ are called the *top* and *bottom layers*, respectively.

A *terminal* is a vertex of G located in the top or bottom layer. A *net* is a set of terminals to be connected. A net containing k terminals is called a k-*net*. The object of the 3-D channel routing problem is to connect the terminals in each net with a Steiner tree (wire) in G using as few layers as possible and as short wires as possible in such a way that Steiner trees spanning distinct nets are vertex-disjoint. A set of nets is said to be *routable* in G if G has vertex-disjoint Steiner trees spanning the nets.

This paper considers the complexity of the following decision problem.

3-D CHANNEL ROUTING

Instance: Positive integers W, D, H, a set of terminals $T \subseteq \{(x, y, z)|x \in [W], y \in$

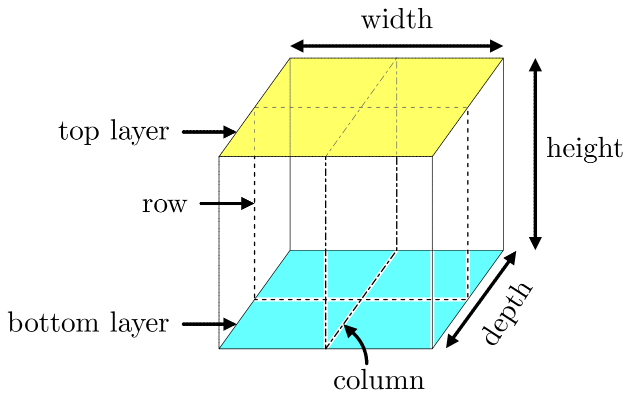


Figure 1: 3-D channel.

$[D], z \in \{1, H\}\}$ and a partition of T into nets $N_1, N_2, \ldots, N_\nu$.

QUESTION: Is a set of nets $\{N_1, N_2, \ldots, N_\nu\}$ routable in a (W, D, H)-channel?

We have two well-known problems as subproblems of 3-D CHANNEL ROUTING, namely, PLANAR CHANNEL ROUTING and TWO-ROW CHANNEL ROUTING. These problems can be stated as follows.

PLANAR CHANNEL ROUTING

INSTANCE: Positive integers W, H, a set of terminals $T \subseteq \{(x, 1, z) | x \in [W], z \in \{1, H\}\}$ and a partition of T into nets $N_1, N_2, \ldots, N_\nu$.

QUESTION: Is a set of nets $\{N_1, N_2, \ldots, N_\nu\}$ routable in a $(W, 1, H)$-channel?

TWO-ROW CHANNEL ROUTING

INSTANCE: Positive integers W, H, a set of terminals $T \subseteq \{(x, 1, z) | x \in [W], z \in \{1, H\}\}$ and a partition of T into nets $N_1, N_2, \ldots, N_\nu$.

QUESTION: Is a set of nets $\{N_1, N_2, \ldots, N_\nu\}$ routable in a $(W, 2, H)$-channel?

It should be noted that TWO-ROW CHANNEL ROUTING has been known as "UNRESTRICTED" TWO-LAYER CHANNEL ROUTING in the literature. The complexity of TWO-ROW CHANNEL ROUTING is a longstanding open question posed by Johnson [7], while PLANAR CHANNEL ROUTING can be solved in polynomial time as shown by Dolev, Karplus, Siegel, Strong, and Ullman [5].

The purpose of this paper is to show the following.

Theorem 1 *3-D CHANNEL ROUTING is $\mathcal{NP}$-complete even for 2-nets.* ∎

We prove Theorem 1 by showing that 3-D CHANNEL ROUTING is $\mathcal{NP}$-hard even for 2-nets in Section 2, and that 3-D CHANNEL ROUTING is in $\mathcal{NP}$ in Section 3.

The complexity of TWO-ROW CHANNEL ROUTING is still open. Also, the complexity of the following problem is open for any fixed integer $k \geq 2$.

2.5-D CHANNEL ROUTING

INSTANCE: Positive integers W, H, a set of terminals $T \subseteq \{(x,y,z)|x \in [W], y \in [k], z \in \{1,H\}\}$ and a partition of T into nets $N_1, N_2, \ldots, N_\nu$.

QUESTION: Is a set of nets $\{N_1, N_2, \ldots, N_\nu\}$ routable in a (W,k,H)-channel?

2 3-D CHANNEL ROUTING is $\mathcal{NP}$-Hard

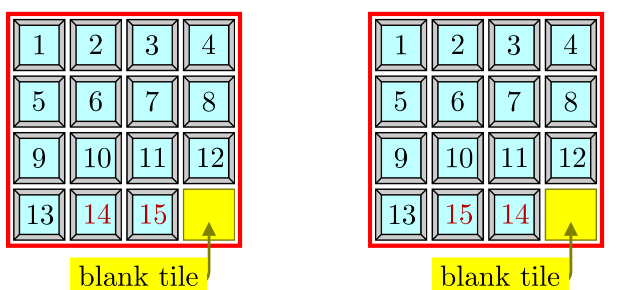
We show in this section the following theorem.

Theorem 2 *3-D CHANNEL ROUGIN is $\mathcal{NP}$-hard even for 2-ntes.* ■

The 3-D channel routing for 2-nets is closely related to the $(r \times s - 1)$-puzzle defined below.

2.1 $(r \times s - 1)$-PUZZLE

The $(r \times s - 1)$-puzzle is a generalization of the well-known 15-puzzle [9]. The $(r \times s - 1)$*-puzzle* is played on an $r \times s$ board, $r, s \geq 2$. There are rs distinct tiles on the board: one *blank tiles* and $rs - 1$ tiles numbered from 1 to $rs - 1$. Each of the rs square locations of the board is occupied by exactly one tile. An instance of $(r \times s - 1)$-puzzle consists of two board configurations C (the *initial configuration*) and C' (the *final configuration*). A *move* is an exchange of the blank tile with a nonblank tile located on a horizontally or vertically adjacent location. The goal of the puzzle is to find a sequence of moves that transforms C to C'. The configuration C' is said to be *reachable* from C if there exists such a sequence of moves. Notice that C' is reachable from C if and only if C is reachable from C'. The configurations C and C' are said to be *reachable* with k moves if there exists a sequence of at most k moves that transforms C to C'. Figure 2 shows



(a) Initial configuration. (b) Final configuration.

Figure 2: Unreachable configurations of $(4 \times 4 - 1)$-puzzzle.

two unreachable configurations of $(4 \times 4 - 1)$-puzzle. This is the original 15-puzzle of Loyd [9]. Our problem is to find a shortest sequence of moves that transforms C to C' if C and C' are reachable. The corresponding decision problem is described as follows.

$(r \times s - 1)$-PUZZLE

INSTANCE: Two $r \times s$ board configurations C and C', and a positive integer k.

QUESTION: Are C and C' reachable with k moves?

Ratner and Warmuth [13] showed the following.

Theorem I *$(r \times s - 1)$-PUZZLE is $\mathcal{NP}$-complete.* ■

While it is intractable to find a shortest sequence of moves that transforms a configuration to another, we can decide in polynomial time if a given two configurations are reachable, and we can find a sequence of a polynomial number of moves that transforms a configuration to another if they are reachable. Let C and C' be any configurations. We assume without loss of generality that the blank tile is located at the right bottom position in the both configurations. Let π be a permutation on the nonblank tiles such that $\pi(t) = t'$ if and only if the location of a nonblank tile t in C is the same as the location of a nonblank tile t' in C'. Wilson [17] showed the following.

Theorem II *Configurations C and C' are reachable if and only if π is even.* ■

The following theorem was shown by Parberry [12] and Kornhauser, Miller, and Spirakis [8].

Theorem III *In an $(n \times n - 1)$-puzzle, a configuration can be transformed to any reachable configuration with $\mathcal{O}(n^3)$ moves.* ■

Theorem III can be easily generalized as follows.

Theorem 3 *In an $(r \times s - 1)$-puzzle, a configuration can be transformed to any reachable configuration with $\mathcal{O}(r^2 s + r s^2)$ moves.* ■

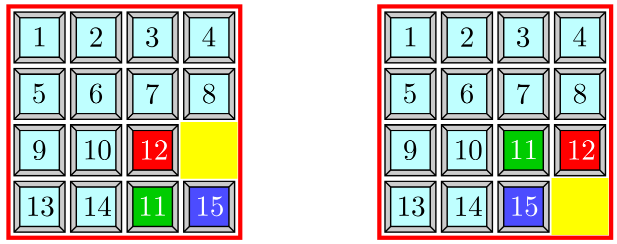
2.2 Proof of Theorem 2

The $(r \times s - 1)$-puzzle is naturally associated with a 3-D channel routing for 2-nets as follows. The configurations C and C' are corresponding to the top and bottom layers. A terminal is corresponding to a location of a nonblank tile on C or C'. A pair of locations of a nonblank tile on C and C' is corresponding to a 2-net. For a sequence of moves that transforms C to C', locations in the sequences for a nonblank tiles corresponds to a part of the wire connecting the terminals of the corresponding 2-net. It is easy to see that each of the first and last moves induces exactly one bend and each of other moves induces exactly two bends. Thus we obtain the following.

Theorem 4 *Configurations C and C' of $(r \times s - 1)$-puzzle are reachable with k moves for $k \geq 2$ if and only if the 2-nets corresponding to the nonblank tiles are routable in an (r, s, k)-channel with at most $2k - 2$ bends.* ■

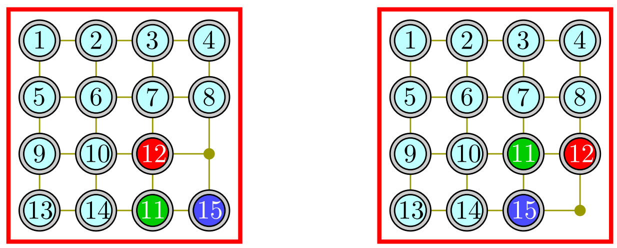
Theorem 4 implies a polynomial time reduction from $(r \times s - 1)$-PUZZLE to 3-D CHANNEL ROUGING. Thus we conclude that 3-D CHANNEL ROUTING is $\mathcal{NP}$-hard by Theorem I. This completes the proof of Theorem 2.

Example 1 *For initial and final configurations C_1 and C_2 of $(4 \times 4 - 1)$-puzzle shown in Fig. 3, the corresponding 2-nets are shown in Fig. 4. A sequence of 3 moves that transforms C_1 to C_2, and the corresponding 3-D channel routing with height 3 are shown in Fig. 5.*



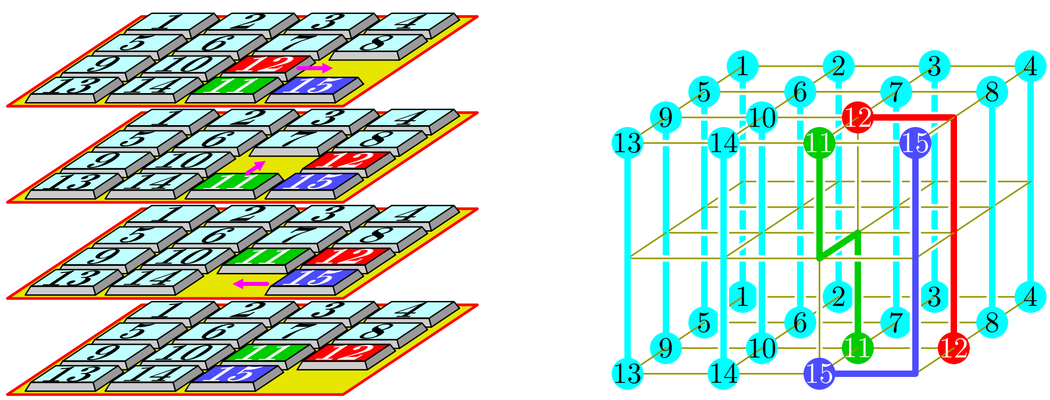
(a) C_1. (b) C_2.

Figure 3: Initial and final configurations of $(4 \times 4 - 1)$-puzzle.



(a) Top layer. (b) Bottom layer.

Figure 4: Corresponding 2-nets.



(a) Sequence of 3 moves that transforms C_1 to C_2.

(b) Corresponding 3-D channel routing with height 3 and 4 bends.

Figure 5: Correspondence between $(4 \times 4 - 1)$-puzzle and 3-D channel routing.

3 3-D CHANNEL ROUTING is in $\mathcal{NP}$

We show in this section the following theorem.

Theorem 5 *3-D CHANNEL ROUTING is in $\mathcal{NP}$.* ■

It should be noted that Theorem 5 is not trivial. Let $\tau = |T|$ and $\tau_i = |N_i|$, $i \in [\nu]$. Since $\mathcal{N} = (N_1, N_2, \ldots, N_\nu)$ is a partition of T, $\tau = \sum_{i=1}^{\nu} \tau_i \geq \nu$. The size of representing each terminal is $\mathcal{O}(\log(WD))$, and the size of representing net N_i is $\mathcal{O}(\tau_i \log \tau)$. Thus the size of an instance of the problem is

$$\begin{aligned}
&\mathcal{O}\left(\log(WDH) + \tau\log(WD) + \sum_{i=1}^{\nu} \tau_i \log \tau\right) \\
&= \mathcal{O}\left(\log(WDH) + \tau\log(WD) + \tau\log\tau\right) \\
&= \mathcal{O}\left(\tau\log(WD\tau) + \log H\right).
\end{aligned}$$

Suppose $\mathcal{N}$ is routable in the (W, D, H)-channel G, and let S_i be a rectilinear Steiner tree connecting the terminals in net N_i, and $\mathcal{S} = \{S_1, S_2, \ldots, S_\nu\}$. Each S_i can be represented by the coordinates of terminals of N_i, Steiner points, and bends of S_i, and edges of S_i. It should be noted that the number of Steiner points of S_i is at most $\tau_i - 1$. Let β_i be the number of bends of S_i, and $\beta = \sum_{i=1}^{\nu} \beta_i$. Then the size of representing S_i is $\Omega(\beta_i)$ and so the size of representing $\mathcal{S}$ is

$$\Omega\left(\sum_{i=1}^{\nu} \beta_i\right) = \Omega(\beta).$$

Since β can be as large as $\Theta(WDH)$, the size of representing $\mathcal{S}$ can be an exponential in the instance size if τ is a polynomial in $\log(WDH)$.

On the other hand, the size of representing S_i is

$$\mathcal{O}\Big((\tau_i + \beta_i)\Big(\log(WDH) + \log(\tau + \beta)\Big)\Big),$$

and so the size of representing $\mathcal{S}$ is

$$\begin{aligned}
&\mathcal{O}\left(\sum_{i=1}^{\nu}\Big((\tau_i + \beta_i)\Big(\log(WDH) + \log(\tau + \beta)\Big)\Big)\right) \\
&= \mathcal{O}\Big((\tau + \beta)\log\big(WDH(\tau + \beta)\big)\Big)
\end{aligned}$$

Thus, in order to prove Theorem 5, it suffices to show that if $\mathcal{N}$ is routable in G, there exists a routing for $\mathcal{N}$ such that β is bounded by a polynomial in the instance size.

3.1 Preliminaries

An $(r \times s - 2)$-*puzzle* has two blank tiles and $rs - 2$ tiles numbered from 1 to $rs - 2$. Contrary to the $(r \times s - 1)$-puzzle, any two configurations in the $(r \times s - 2)$-puzzle are reachable as shown in [8]. It is easy to see the following two theorems.

Theorem 6 *In an $(r \times s - 2)$-puzzle, a configuration is reachable to any configuration with $\mathcal{O}(r^2 s + rs^2)$ moves.* ■

Theorem 7 *If two configurations of $(r \times s - 2)$-puzzle are reachable with k moves then the 2-nets corresponding to the nonblank tiles are routable in an (r, s, k)-channel with at most $2k - 2$ bends.* ■

Now we are ready to prove the following.

Theorem 8 *If a set of nets $\mathcal{N} = \{N_1, N_2, \ldots, N_\nu\}$ is routable in a (W, D, H)-channel for some finite H, then $\mathcal{N}$ is also routable in a $(W, D, \mathcal{O}(\tau^2))$-channel with $\mathcal{O}(\tau^2)$ bends.*

Proof (Sketch). We distinguish two cases.

Case 1 When each N_i is a 2-net and has terminals both on top and bottom layers: Notice that $\nu = \Theta(\tau)$. We further distinguish four cases.

Case 1-1 When $\nu = WD$: It is easy to see that $\mathcal{N}$ is routable in a 3-D channel of a finite height if and only if the locations of the terminals on the top and bottom layers are same for every 2-net. Thus $\mathcal{N}$ is routable in a 3-D channel of height $\mathcal{O}(1)$ with no bend.

Case 1-2 When $\nu = WD - 1$: The routing for this case corresponds to a $(W \times D - 1)$-puzzle. Thus, if $\mathcal{N}$ is routable in a (W, D, H)-channel, then $\mathcal{N}$ is also routable in a $(W, D, \mathcal{O}(W^2D + WD^2))$-channel with $\mathcal{O}(W^2D + WD^2)$ bends by Theorems 3 and 4. Since $WD = \nu + 1 = \mathcal{O}(\tau)$, $\mathcal{N}$ is routable in a $(W, D, \mathcal{O}(\tau^2))$-channel with $\mathcal{O}(\tau^2)$ bends.

Case 1-3 When $\nu = WD - 2$: The routing for this case can be reduced to a $(W \times D - 2)$-puzzle. Thus, $\mathcal{N}$ is routable in a $(W, D, \mathcal{O}(W^2D + WD^2))$-channel with $\mathcal{O}(W^2D + WD^2)$ bends by Theorems 6 and 7. Since $WD = \nu + 2 = \mathcal{O}(\tau)$, $\mathcal{N}$ is routable in a $(W, D, \mathcal{O}(\tau^2))$-channel with $\mathcal{O}(\tau^2)$ bends.

Case 1-4 When $\nu \leq WD - 3$: Let $W' = \lceil (\nu + 2)/D \rceil$, and $\eta = W'D - 2$. Notice that $\eta = \mathcal{O}(\tau)$. By adding $\eta - \nu$ dummy 2-nets, each of which has one terminal on the top layer and the other on the bottom layer, we have a (W, D, H)-channel with a set of 2-nets, $\mathcal{N}' = \{N_1, N_2, \ldots, N_\eta\}$, where $N_{\nu+1}, N_{\nu+2}, \ldots, N_\eta$ are the dummy 2-nets. For $i \in [\eta]$, let $t_i^{\langle 1 \rangle}$ and $t_i^{\langle 2 \rangle}$ be the terminals of 2-net N_i on the top and bottom layers, respectively, and $L^{\langle 1 \rangle}$ and $L^{\langle 2 \rangle}$ be two layers in the channel. (See Fig. 6.) Then, we can locate virtual terminals $\left\{v_1^{\langle i \rangle}, v_2^{\langle i \rangle}, \ldots, v_\eta^{\langle i \rangle}\right\}$ on $L^{\langle i \rangle}$, $i \in [2]$, at vertices with x-coordinate at most W' so that a set of virtual 2-nets $\left\{\{t_j^{\langle i \rangle}, v_j^{\langle i \rangle}\} | j \in [\eta]\right\}$ is routable in a (W, D, η)-channel with $\mathcal{O}(\eta)$ bends, $i \in [2]$. (The details are omitted in this extended abstract, due to space limitation.) Moreover, a set of virtual 2-nets $\left\{\{v_j^{\langle 1 \rangle}, v_j^{\langle 2 \rangle}\} | j \in [\eta]\right\}$ is routable in a $(W', D, \mathcal{O}(\eta^2))$-channel with $\mathcal{O}(\eta^2)$ bends, since this is reduced to Case 1-3. Combining the routings for virtual 2-nets $\{t_j^{\langle 1 \rangle}, v_j^{\langle 1 \rangle}\}$, $\{v_j^{\langle 1 \rangle}, v_j^{\langle 2 \rangle}\}$, and $\{v_j^{\langle 2 \rangle}, t_j^{\langle 2 \rangle}\}$ for each $j \in [\eta]$, we obtain the routing for $\mathcal{N}'$ in a $(W, D, \mathcal{O}(\tau^2))$-channel with $\mathcal{O}(\tau^2)$ bends, since $\eta = \mathcal{O}(\tau)$.

Case 2 General case: We can easily extend the proof for 2-nets in Case 1 to the general case, but the details are omitted in this extended abstract, due to space limitation. ■

3.2 Proof of Theorem 5

It suffices to show the following.

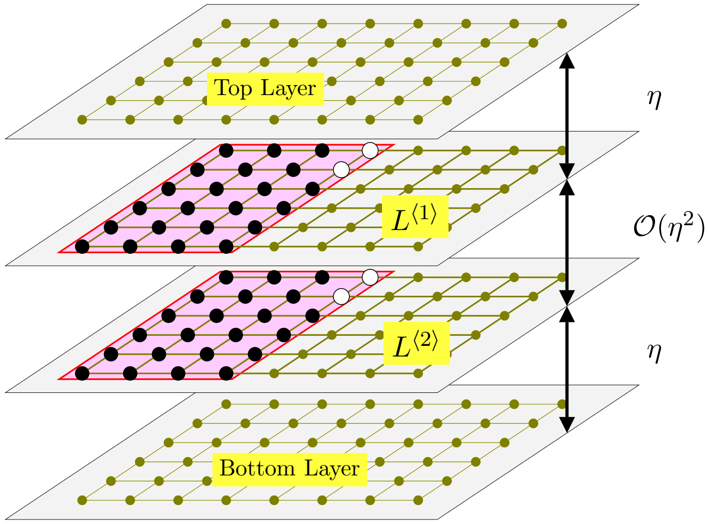


Figure 6: Strategy of the 3-D routing when $\nu \leq WD - 3$.

Theorem 9 *If a set of nets $\mathcal{N} = \{N_1, N_2, \ldots, N_\nu\}$ is routable in a (W, D, H)-channel G, then $\mathcal{N}$ is also routable in G with $\mathcal{O}(\tau^8)$ bends.*

Proof. By Theorem 8, there exists a function $f(\tau)$ of $\mathcal{O}(\tau^2)$ such that if $\mathcal{N}$ is routable in a (W, D, H)-channel then $\mathcal{N}$ is also routable in a $(W, D, f(\tau))$-channel with $\mathcal{O}(\tau^2)$ bends. We distinguish two cases.

Case 1 $H \geq f(\tau)$: It is clear that $\mathcal{N}$ is routable in G with $\mathcal{O}(\tau^2)$ bends, and we are done.

Case 2 $H < f(\tau)$: Let $\varphi_1, \varphi_2, \ldots, \varphi_\lambda$ and $\psi_1, \psi_2, \ldots, \psi_{\lambda'}$ be the increasing sequences of x- and y-coordinates of terminals, respectively. By definition, a terminal is located at (φ_i, ψ_j, z), for some $i \in [\lambda]$, $j \in [\lambda']$, $z \in \{1, H\}$, and $\lambda, \lambda' \leq \tau$. Let $\varphi_0 = \psi_0 = 1$, $\varphi_{\lambda+1} = W$, and $\psi_{\lambda'+1} = D$.

Since $\mathcal{N} = \{N_1, N_2, \ldots, N_\nu\}$ is routable in G, there exist vertex-disjoint Steiner trees S_i for N_i, $1 \leq i \leq \nu$. Let $\mathcal{S} = \{S_1, S_2, \ldots, S_\nu\}$. For each $m \in [\lambda + 1]$, let G^X_m be a subgrid of G induced by the vertices in $\{(x, y, z) | \varphi_{m-1} \leq x \leq \varphi_m, y \in [D], z \in [H]\}$. If $\varphi_m - \varphi_{m-1} \geq f(\tau)$, we can partially reroute $\mathcal{S}$ in G^X_m so that the number of bends in G^X_m is $\mathcal{O}(\tau^2)$ by Theorem 8. Here, the columns defined by $x = \varphi_{m-1}$ and $x = \varphi_m$ are considered as the top and bottom layers of a 3-D channel.

Similarly, for each $m' \in [\lambda' + 1]$, let $G^Y_{m'}$ be a subgrid of G induced by the vertices in $\{(x, y, z) | x \in [W], \psi_{m'-1} \leq y \leq \psi_{m'}, z \in [H]\}$. If $\psi_{m'} - \psi_{m'-1} \geq f(\tau)$, we can partially reroute $\mathcal{S}$ in $G^Y_{m'}$ so that the number of bends in $G^Y_{m'}$ is $\mathcal{O}(\tau^2)$.

For a subgraph H of G, $G \setminus H$ denotes the induced subgraph of G on $V(G) - V(H)$, i.e., the graph obtained from G by deleting all vertices in $V(H)$ and all edges incident to at least one vertex of $V(H)$. For two graphs H_1 and H_2, $H_1 \cup H_2$ denotes the graph with vertex set $V(H_1) \cup V(H_2)$ and edge set $E(H_1) \cup E(H_2)$. Similarly, $H_1 \cap H_2$ denotes the graph with vertex set $V(H_1) \cap V(H_2)$ and edge set $E(H_1) \cap E(H_2)$.

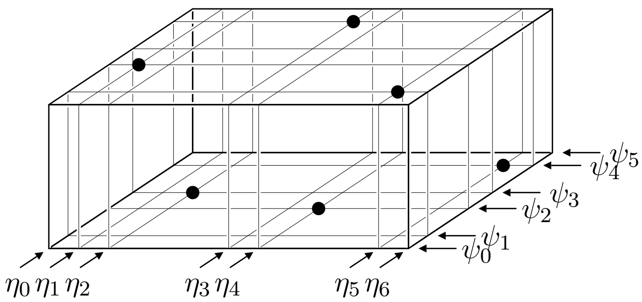
Let

$$\mathcal{X} \;=\; \bigcup \left\{ G^X_m | \varphi_m - \varphi_{m-1} + 1 \geq f(\tau) \right\},$$

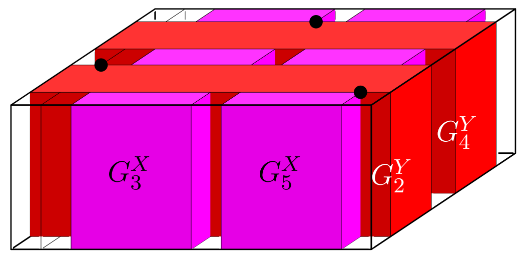
$$\mathcal{Y} \;=\; \bigcup \left\{ G^Y_{m'} | \psi_{m'} - \psi_{m'-1} + 1 \geq f(\tau) \right\},$$

$\overline{\mathcal{X}} = G \setminus \mathcal{X}$, and $\overline{\mathcal{Y}} = G \setminus \mathcal{Y}$. After the reroutings, the total number of bends in $\mathcal{X} \cup \mathcal{Y}$ is $\mathcal{O}(\lambda\tau^2 + \lambda'\tau^2) = \mathcal{O}(\tau^3)$. Moreover, the number of bends in $\overline{\mathcal{X}} \cap \overline{\mathcal{Y}}$ can be bounded by the number of grid points $|V(\overline{\mathcal{X}} \cap \overline{\mathcal{Y}})| = \mathcal{O}(\lambda\lambda' f(\tau)^2 H) = \mathcal{O}(\tau^8)$. Thus, we conclude that $\mathcal{N}$ is routable with $\mathcal{O}(\tau^8)$ bends. This completes the proof of Theorem 9. ■

Example 2 *Figure 7 (b) illustrates an example of $G^X_3 \cup G^X_5 \cup G^Y_2 \cup G^Y_4$ for the terminals shown in Fig. 7 (a).*



(a) η_i and ψ_j in G.



(b) $G^X_3 \cup G^X_5 \cup G^Y_2 \cup G^Y_4$.

Figure 7: x- and y-coordinates, and subgrids of G.

4 Concluding Remarks

The Manhattan model is one of the most popular 2-D channel routing models for practitioners. Szymanski [14] proved that a decision problem of MANHATTAN 2-D CHANNEL ROUTING is $\mathcal{NP}$-complete, while the complexity of the problem for 2-nets has been open as mentioned in [10]. It is interesting to note that 3-D CHANNEL ROUTING is $\mathcal{NP}$-complete even for 2-nets as we have shown in this paper.

It is worth noting that if the layers are square 2-D grids of area 4ν, the terminals are located on vertices with even x- and y-coordinates, and each net has terminals both on top and bottom layers, then any set of ν 2-nets is routable with height $\mathcal{O}(\sqrt{\nu})$ and $\mathcal{O}(\nu)$ bends [15]. Moreover, there is a set of nets that requires 3-D channel of height $\Omega(\sqrt{\nu})$ to be routed [15].

References

[1] J. Baliga, "Chips go vertical," IEEE Spectrum, vol.41, no.3, pp.43–47, 2004.

[2] K. Banerjee, S.J. Souri, P. Kapur, and K.C. Saraswat, "3-D ICs: A novel chip design for improving deep-submicrometer interconnect performance and systems-on-chip integration," Proc. of the IEEE, vol.89, no.5, pp.602–633, 2001.

[3] M.L. Brady, D.J. Brown, and P.J. McGuinness, “The three-dimensional channel routing problem,” in Algorithmic Aspects of VLSI Layout, pp.213–244, World Scientific, 1993.

[4] S. Das, A. Fan, K.N. Chen, C.S. Tan, N. Checka, and R. Reif, “Technology, performance, and computer-aided design of three-dimensional integrated circuits,” Proc. ISPD, pp.92–98, 2004.

[5] D. Dolev, K. Karplus, A. Siegel, A. Strong, and J. Ullman, “Optimal wiring between rectangles,” Proc. 13th Ann. ACM Symposium on Theory of Computing, pp.312–317, 1981.

[6] R. Enbody, G. Lynn, and K. Tan, “Routing the 3-D chip,” Proc. the 28-th Design Automation Conference DAC’91, 1991.

[7] D. Johnson, “The NP-completeness column: An ongoing guide,” Journal of Algorithms, pp.381–395, 1982.

[8] D. Kornhause, G. Miller, and P. Spirakis, “Coodinating pebble motion on graphs, the diameter of permutation groups, and applications,” 25th Annual Symposium on Foundation of Computer Science (FOCS’84), pp.241–250, 1984.

[9] S. Loyd, Mathematical Puzzles of Sam Loyd, Dover, New York, 1959.

[10] R. Möhring, D. Wagner, and F. Wagner, VLSI Network Design, ch. 8, In: M.O. Ball, T.L. Magnanti, C.L. Monma, and G.L.Nemhauser, editors, *Handbooks in Operations Research*, North-Holland, 1995.

[11] S.T. Obenaus and T.H. Szymanski, “Gravity: Fast placement for 3-D VLSI,” ACM Trans. Design Automation of Electronic Systems, vol.8, no.3, pp.298–315, 2003.

[12] I. Parberry, “A real-time algorithm for the $(n^2 - 1)$-puzzle,” vol.56, pp.23–28, 1995.

[13] D. Ratner and M. Warmuth, “The (n^2-1)-puzzle and related relocation problems,” Journal of Symbolic Computation, vol.10, pp.111–137, 1990.

[14] T.G. Szymanski, “Dogleg channel routing is NP-complete,” IEEE Trans. Computer-Aided Design, vol.Cad-4, no.1, pp.31–41, 1985.

[15] S. Tayu, P. Hurtig, Y. Horikawa, and S. Ueno, “On the three-dimensional channel routing,” Proc. IEEE International Symposium on Circuits and Systems ISCAS’05, pp.180–183, 2005.

[16] C.C. Tong and C.L. Wu, “Routing in a three-dimensional chip,” IEEE Trans. Computers, vol.44, no.1, pp.106–117, 1995.

[17] R.M. Wilson, “Graph puzzles, homotopy, and the alternating group,” Journal of Combinatorial Theory, vol.B, no.16, pp.86–96, 1974.