A Note on the Energy-Aware Mapping for NoCs

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Abstract— The mapping problem for NoCs is to decide how to assign the tasks of an application onto the PEs of a network such that some objective function is optimized. The mapping problem is one of the most fundamental problems on the design of NoCs, since the application mapping onto the network greatly impacts both the performance and energy consumption of the NoC. It has been known that the energy-aware mapping problem is NP-hard even if an application graph is a caterpillar with vertex degree at most 4 and a network graph is a square mesh.

This paper considers a special case of the problem which is solvable in polynomial time. We show that if an application graph is a caterpillar with vertex degree at most 3 and a network graph is a ladder, the problem can be solved in linear time.

I. INTRODUCTION

As the number of components on a single chip increases, the design of the communication architecture plays an important role in defining the area, performance, and energy consumption of the system. To overcome the complex communication problems that arise as the number of on-chip components increases, network-on-chip (NoC) architectures have been proposed to replace global interconnects [2]. The mapping problem for NoCs is to decide how to assign the tasks of the application onto the PEs of the network such that the metrics of interest are optimized. The mapping problem is one of the most fundamental problems on the design of NoCs, since the application mapping onto the network greatly impacts both the performance and energy consumption of the NoC [7]. The energy-aware mapping for NoCs has been considered in the literature. Since the problem is known to be NP-hard [10], many heuristic algorithms have been proposed [1], [3]–[6], [8], [12]–[14]. For some practical applications, the optimal mappings onto mesh-based NoCs are shown in [9].

This paper proposes in Section III a linear time algorithm for the energy-aware mapping problem if an application graph is a caterpillar with vertex degree at most 3 and a network graph is a ladder.

II. ENERGY-AWARE MAPPING

The application graph G_A of an application A is a graph such that each vertex represents a task in A, and each edge $(u, v) \in E(G_A)$ represents a data dependency between two tasks u and v. The amount of data transfers between tasks uand v is represented by a weight function

$$w: E(G_A) \to \mathbb{Z}^+$$

The network graph M is a graph such that each vertex $p \in V(M)$ represents a PE, and each edge $(p,q) \in E(M)$ represents a direct communication between two PEs p and q.

Given G_A and M, the mapping problem is to find a one-to-one mapping

$$\phi: V(G_A) \to V(M)$$

such that some objective function is optimized.

In the energy-aware mapping problem, we assume a minimal routing, i.e., communication data transfers between u and v is executed along a shortest path in M between $\phi(u)$ and $\phi(v)$. The length of such a path is called the *dilation* of (u, v), and denoted by $dil_{\phi}(u, v)$. It is shown in [4] that the total communication energy is proportional to

$$(\phi) = \sum_{(u,v)\in E(G_A)} w(u,v) \operatorname{dil}_{\phi}(u,v),$$

if we employ a routing technique which is deterministic, deadlock-free, minimal, and wormhole-based. Thus, the energy-aware mapping problem is to find a mapping ϕ with minimum $\mathcal{E}(\phi)$. The decision version of the energy-aware mapping problem can be described as follows.

ENERGY-AWARE MAPPING

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Instance. Application graph G_A , network graph M, weight function $w : E(G_A) \to \mathbb{Z}^+$, and integer bound $K \in \mathbb{Z}^+$.

Question. Is there a one-to-one mapping $\phi : V(G_A) \rightarrow V(M)$ such that $\mathcal{E}(\phi) \leq K$?

A tree *C* is called a *caterpillar* if the vertices of degree at least 2 are on a path in *C*. A longest path in *C* is called a *spine* of *C*. It should be noted that a spine contains all vertices of degree at least 2. A caterpillar is said to be *one-legged* if the maximum degree of a vertex is at most 3. An $m \times n$ mesh $M_{m,n}$ is a graph with $V(M_{m,n}) = \{(i,j) \mid i \in [m], j \in [n]\}$, and vertices (i, j) and (i', j') are connected by an edge if and only if |i - i'| + |j - j'| = 1, where $[l] = \{1, 2, \dots, l\}$. For a vertex $(i, j) \in V(M_{m,n})$, *i* and *j* are called the *x*- and *ycoordinates* of (i, j), respectively. An $m \times n$ mesh is said to be *optimal* for a graph *G* if

$$\max\{n(m-1), m(n-1)\} < |V(G)| \le nm.$$

The following is shown in [11].

Theorem I: ENERGY-AWARE MAPPING is NP-complete even if application graph G_A is a caterpillar with maximum degree 4 and a network graph M is an $N \times 4$ optimal mesh or optimal square mesh.

III. A SPECIAL CASE

We consider a case in which G_A is a one-legged caterpillar with 2n vertices, M is an $n \times 2$ mesh, the weight of an edge of G_A is a constant c, and K = (2n - 1)c.

Before stating our main results, we need some preliminaries. Let C be a one-legged caterpillar. A partition

$$(V_1, V_2, \ldots, V_k)$$

of V(C) is called an *even-permutation* of C if the following conditions are satisfied:

- 1) $|V_j|$ is even for any $j \in [k]$,
- 2) If $(u,v) \in E(C)$ then $\{u,v\} \subseteq V_j \cup V_{j+1}$ for some $j \in [k-1]$,
- C[V_j], a subgraph of C induced by V_j, is connected for any j ∈ [k],
- For any e ∈ E(C[V_j]), C[V_j] − e, a graph obtained from C[V_j] by deleting e, has no connected component with an even number of vertices (j ∈ [k]).

We call V_j a *block* of the even-partition. It is easy to see that there exists exactly one edge $(u, v) \in E(C)$ such that $u \in V_j$ and $v \in V_{j+1}$ for any $j \in [k-1]$, and such (u, v) is an edge of a spine. For a spine S of C a vertex $p \notin V(S)$ is called a *pendant* at s if $(p, s) \in E(C)$. The following two lemmas are immediate from the definition.

Lemma 1: C has an even-partition if and only if |V(C)| is even.

Lemma 2: If p is a pendant at s then p and s are in the same block. That is, $\{p, s\} \subseteq V_j$ for some $j \in [k]$.

Lemma 3: The even-partition of C is unique.

Proof: Let S be the spine of C such that

$$V(S) = \{s_1, s_2, \dots, s_l\} \text{ and } E(S) = \{(s_i, s_{i+1}) \mid i \in [l-1]\}$$

Suppose contrary that C has distinct even-partitions (V_1, V_2, \ldots, V_k) and (U_1, U_2, \ldots, U_l) such that both V_1 and U_1 contain s_1 . Let m be the smallest integer such that $V_m \neq U_m$. Without loss of generality we assume that $V_m \subset U_m$. Let $L_i = \{s_i, p_i\}$ if there is a pendant p_i at s_i , and $L_i = \{s_i\}$ otherwise. From Lemma 2, $V_m = \bigcup_{i=i_1}^{i_2} L_i$ and $U_m = \bigcup_{i=i_1}^{i_3} L_i$ for some $i_1 \leq i_2 < i_3$. Since $|\bigcup_{i=i_2+1}^{i_3} L_i| = |\bigcup_{i=i_1}^{i_3} L_i| - |\bigcup_{i=i_1}^{i_2} L_i|$ is even, deleting (s_{i_2}, s_{i_2+1}) from $C[U_m]$ results in the two connected components having an even number of vertices, contradicting to 4) of the definition of the even-partition. Thus, we have the lemma.

For any integer $i \ge 0$, let Γ_i be a caterpillar with 2 + 2i vertices, and *i* vertices of degree 3. Γ_0 , Γ_1 , and Γ_2 are shown in Fig. 1(a), (b), and (c), respectively.

Lemma 4: For the even-partition (V_1, V_2, \ldots, V_k) of C, $C[V_j], j \in [k]$, is isomorphic to Γ_i for some *i*.

Proof: It suffices to show that the degree of every vertex in $C[V_j]$ is 1 or 3. Let S be the spine of C such that $V(S) = \{s_1, s_2, \ldots, s_l\}$ and $E(S) = \{(s_i, s_{i+1}) \mid i \in [l-1]\}$. Let $L_i = \{s_i, p_i\}$ if there is a pendant at s_i , and $L_i = \{s_i\}$ otherwise.

Suppose contrary that there exists $s_{i_0} \in V_m$ such that the degree of s_{i_0} in $C[V_m]$ is 2. Let i_1 and i_2 be integers such that $\bigcup_{j=i_1}^{i_2} = V_m$. If $|L_{i_0}| = 2$, $i_0 = i_1$ or i_2 then both $|L_{i_0}|$ and $|V_m - L_{i_0}|$ are even, contradicting to 4) of the definition





Fig. 2. Examples of one-legged caterpillars.

of the even-partition. If $|L_{i_0}| = 1$, $i_1 < i_0 < i_2$ then both $|\bigcup_{j=i_1}^{i_0} L_j|$ and $|\bigcup_{j=i_0+1}^{i_2} L_j|$ are even, or both $|\bigcup_{j=i_1}^{i_0-1} L_j|$ and $|\bigcup_{j=i_0}^{i_2} L_j|$ are even, contradicting to 4) too. Thus, we have the lemma.

An even-partition (V_1, V_2, \ldots, V_k) of C is called a π partition of C if $C[V_j]$ is isomorphic to Γ_0 or Γ_2 for any $j \in [k]$.

Fig. 2(a) shows an example of a one-legged caterpillar C with a π -partition, and Fig. 2(b) shows the subgraphs induced by blocks of the π -partition.

Now, we are ready to state our main results. The following is proved in the next section.

Theorem 1: Let G_A be a one-legged caterpillar with 2n vertices, and M be an $n \times 2$ mesh. Then, there exists a mapping ϕ of G_A onto M with $\operatorname{dil}_{\phi}(u, v) = 1$ for any $(u, v) \in E(G_A)$ if and only if G_A has a π -partition. Moreover, a π -partition can be found in linear time, if any.

We obtain the following from Theorem 1.

Theorem 2: ENERGY-AWARE MAPPING can be solved in linear time if G_A is a one-legged caterpillar with 2nvertices, M is an $n \times 2$ mesh, the weight of an edge of G_A is a constant c, and K = (2n - 1)c.

Proof: If G_A is a 2n vertex graph, M is an $n \times 2$ mesh, the weight of an edge of G_A is a constant c, and K = (2n - 1)c, then $\phi: V(G_A) \to V(M)$ is a mapping with $\mathcal{E}(\phi) \leq K$ if and only if $\operatorname{dil}_{\phi}(u, v) = 1$ for any $(u, v) \in E(G_A)$. Thus, we have the theorem from Theorem 1.

IV. PROOF OF THEOREM 1

It is easy to see that a π -partition can be found in linear time, if any. Thus it suffices to show the following two lemmas. Let C be a one-legged caterpillar with 2n vertices.

Lemma 5: If C has a π -partition, then there exists a mapping ψ of C onto $M_{n,2}$ such that $\operatorname{dil}_{\psi}(u,v) = 1$ for any $(u,v) \in E(C)$.

Lemma 6: If there exists a mapping ψ of C onto $M_{n,2}$ such that $\operatorname{dil}_{\psi}(u,v) = 1$ for any $(u,v) \in E(C)$, then C has a π -partition.

A. Proof of Lemma 5

Let C be a one-legged caterpillar with spine S, and

$$(V_1, V_2, \ldots, V_k)$$

be a π -partition of V(C). Let l_h and r_h be the vertices in V_h $(h \in [k])$ such that $(r_h, l_{h+1}) \in E(C)$ $(h \in [k-1])$, where $l_1 \in V_1 \cap V(S)$ and $r_k \in V_k \cap V(S)$ are the vertices of degree 1.

Now, we will construct a desired mapping $\psi : V(C) \rightarrow V(M_{n,2})$. For convenience, we denote by $\psi_x(v)$ and $\psi_y(v)$ the *x*- and *y*-coordinates of $\psi(v)$, respectively. For $j \in \{1, 2\}$, we denote that $\overline{j} = 3 - j$.

A mapping $\psi(v)$ of $v \in V_h$ is defined recursively for h. If h = 1 or $\psi(v)$ (for $\forall v \in V_{h-1}$) is given for $2 \le h \le k$, we define $\psi(v)$ for $\forall v \in V_h$ as follows. Define that $\psi(l_1) = (1, 1)$ if h = 1, and $\psi(l_h) = (\psi_x(r_{h-1} + 1), \psi_y(r_{h-1}))$ if $h \ge 2$.

We have the following three cases. Subgraphs induced by V_h for Cases 1–3 are shown in Fig. 3, where red vertices denote the vertices in V(S):



Fig. 3. Induced subgraphs $C[V_h]$.

Case 1 $C[V_h]$ is isomorphic to Γ_0 and $r_h \neq l_h$: We just define $\psi(r_h) = (\psi_x(l_h), \overline{\psi_x(l_h)})$.

Case 2 $C[V_h]$ is isomorphic to Γ_0 and $r_h = l_h$: We just define $\psi(p) = (\psi_x(l_h), \overline{\psi_x(l_h)})$, where p is the pendant at l_h .

Case 3 $C[V_h]$ is isomorphic to Γ_2 (and $r_h \neq l_h$): Let $V_h = \{l_h, l'_h, l''_h, r_h, r'_h, r''_h\}$, where l_h, l'_h, r'_h , and r_h are vertices of spine appearing in this order, and (l'_h, l''_h) , $(r'_h, r'_h) \in E(C)$. See Fig. 3 (c).

For $v \in V_h$, $\psi(v)$ is defined as

$$\begin{split} \psi_x(l''_h) &= \psi_x(r_h) = \psi_x(l_h) + 2, \\ \psi_x(l'_h) &= \psi_x(r'_h) = \psi_x(l_h) + 1, \\ \psi_x(r''_h) &= \psi_x(l_h), \\ \psi_y(l'_h) &= \psi_y(l''_h) = \psi_y(l_h), \text{ and} \\ \psi_y(r_h) &= \psi_y(r'_h) = \psi_y(r''_h) = \overline{\psi_y(l_h)} \end{split}$$

as shown in Fig. 4(c).

It is easy to see that $\operatorname{dil}_{\psi}(u, v) = 1$ for any $(u, v) \in E(C)$, and we have the lemma



(c) Images of ψ in Case 3.

Fig. 4. Mapping for induced subgraphs $G[V_h]$.

B. Proof of Lemma 6

Let ψ be a mapping of C onto $M_{n,2}$ such that $\operatorname{dil}_{\psi}(u, v) = 1$ for any $(u, v) \in E(C)$. Since $|V(C)| = |V(M_{n,2})| = 2n$, we have the following.

Lemma 7: ψ is a bijection. Since |V(C)| = 2n, C has the even-partition

 (V_1, V_2, \ldots, V_k)

by Lemma 1. We are going to show that the even-partition is a π -partition.

Lemma 8: $C[V_i]$ is not isomorphic to Γ_1 for any $j \in [k]$.

Proof: Suppose contrary that there exists an integer j such that $C[V_j]$ is isomorphic to Γ_1 . Let $V_j = \{c, p, r_j, l_j\}$, where c is adjacent with all of other three vertices.

Without loss of generality, we assume that $\psi_y(c) = 1$. Then, $\{\psi(v) \mid v \in V_h - \{c\}\} = \{(X - 1, 1), (X, 2), (X + 1, 1)\}$ for some integer X. If M_L and M_R are the two connected components of $M_{n,2} - \psi(V_h)$ then $|V(M_L)|$ and $|V(M_R)|$ are odd integers. If $\mathcal{L} = \bigcup_{l=1}^{h-1} V_l$ and $\mathcal{R} = \bigcup_{l=h+1}^{k} V_l$ then $|\mathcal{L}|$ and $|\mathcal{R}|$ are even integers. Thus from Lemma 7, one of the following conditions holds:

(a)
$$\psi(\mathcal{L}) \cap V(M_L) \neq \emptyset$$
 and $\psi(\mathcal{L}) \cap V(M_R) \neq \emptyset$,
(b) $\psi(\mathcal{R}) \cap V(M_L) \neq \emptyset$ and $\psi(\mathcal{R}) \cap V(M_R) \neq \emptyset$.

Without loss of generality, we assume that (a) holds. Let s and t be vertices in \mathcal{L} such that $\psi(s) \in V(M_L)$ and $\psi(t) \in V(M_R)$, and let (v_1, v_2, \ldots, v_k) be the path of C connecting s and t, where $v_1 = s$ and $v_k = t$. Let l be an integer satisfying $\psi(v_l) \in V(M_L)$ and $\psi(v_{l+1}) \in V(M_L)$. Since $\psi_x(v_l) \leq X - 1$ and $\psi_x(v_{l+1}) \geq X + 1$, we have $\operatorname{dil}_{\psi}((v_l, v_{l+1})) \geq 2$, a contradiction. Thus, any $C[V_i]$ is not isomorphic to Γ_1 .

For a graph H and a vertex $v \in V(H)$, $N_H(v)$ is the set of vertices adjacent with v in H. For $U \subseteq V(H)$, we define that $N_H(U) = \bigcup_{v \in U} N_H(v)$. Since $\operatorname{dil}_{\psi}(u, v) = 1$ for any $(u, v) \in E(C)$, we have the following.

Lemma 9: $|N_C(U)| \leq |N_{M_{n,2}}(\psi(U))|$ for any $U \subseteq V(C)$.



Fig. 5. C₃.

Lemma 10: Let $s_i, s_{i+1}, s_{i+2} \in V(C)$ be vertices of degree three such that $(s_i, s_{i+1}), (s_{i+1}, s_{i+2}) \in E(C)$. Then,

$$\begin{aligned} \psi_x(s_i) - \psi_x(s_{i+1}) &= \psi_x(s_{i+1}) - \psi_x(s_{i+2}) = \pm 1, \text{ and} \\ \psi_y(s_i) &= \psi_y(s_{i+1}) = \psi_y(s_{i+2}). \end{aligned}$$

Proof: Let $U = \{s_i, s_{i+1}, s_{i+2}\}$. We have $\psi_x(s_i) \neq \psi_x(s_{i+1})$, for otherwise we have :

$$\begin{array}{rcl} |\psi_x(s_{i+1}) - \psi_x(s_{i+2})| &=& 1, \\ & \psi_y(s_{i+2}) &=& \psi_y(s_{i+1}), \text{ and} \\ & \psi_y(s_i) &=& 3 - \psi_y(s_{i+1}), \end{array}$$

which implies that $|N_{M_{n,2}}(\psi(U))| = 7$, contradicting to Lemma 9, since we have $|N_C(U)| = 8$. By the similar arguments, we also have $\psi_x(s_{i+2}) \neq \psi_x(s_{i+1})$. Since $\psi(s_{i+1}) = (x, y)$ has only three adjacent vertices (x, 3-y), (x-1, y), and (x+1, y), we have $\{\psi(s_i), \psi(s_{i+2})\} = \{(x-1, y), (x+1, y)\}$. Thus, we have the lemma.

Lemma 11: $C[V_j]$ is not isomorphic to Γ_l for any $j \in [k]$ and $l \geq 3$.

Proof: Suppose contrary that $C[V_h]$ is isomorphic to Γ_l for some $h \in [k]$ and $l \geq 3$. Let

$$\begin{split} V_h &= \{s_i \mid 0 \le i \le l+1\} \cup \{p_i \mid 1 \le i \le l\} \text{ and } \\ E(C[V_h]) &= \{(s_i, s_{i+1}) \mid 0 \le i \le l\} \\ & \cup \{(s_i, p_i) \mid 1 \le i \le l\}, \end{split}$$

where s_0 and s_{l+1} are adjacent with $r_{h-1} \in V_{h-1}$ and $l_{h+1} \in V_{h+1}$, respectively, and $(p_i, s_i) \in E(C)$. Define that

$$\mathcal{L} = \bigcup_{i=1}^{h-1} V_i$$
 and
 $\mathcal{R} = \bigcup_{i=h+1}^{k} V_i.$

Let

$$\mathcal{L}^+ = \mathcal{L} \cup \{s_0, s_1, p_1\}, \mathcal{R}^+ = \mathcal{R} \cup \{s_i \mid 3 \le i \le l+1\} \cup \{p_i \mid 3 \le i \le l\}.$$

It should be noted that $(\mathcal{L}^+, \mathcal{R}^+, \{s_2, p_2\})$ is a partition of V(C). See Fig. 5. Thus, $C[\mathcal{L}^+]$ and $C[\mathcal{R}^+]$ are connected subgraphs of C. We assume without loss of generality that

$$\psi_y(s_2) = 1$$
, and
 $\psi_x(s_1) = \psi_x(s_2) - 1$.

Then from Lemma 10,

$$\psi(s_1) = (X - 1, 1)$$
 and
 $\psi(s_3) = (X + 1, 1),$

where $X = \psi_x(s_2)$. Therefore, $\psi(p_2) = (X, 2)$, since $(p_2, s_2) \in E(C)$. Since ψ is a dilation 1 mapping and $C[\mathcal{L}^+]$ is connected, $\psi_x(v) \leq X - 1$ for any $v \in \mathcal{L}^+$. From Lemma 7, we have $|C[\mathcal{L}^+]| = 2X - 2$. However, $|\mathcal{L}^+| = |\mathcal{L}| + 3$ is odd, a contradiction. Thus, we have the lemma.

From Lemmas 1, 8, and 11, we conclude that the evenpartition is a π -partition, and we have Lemma 6.

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